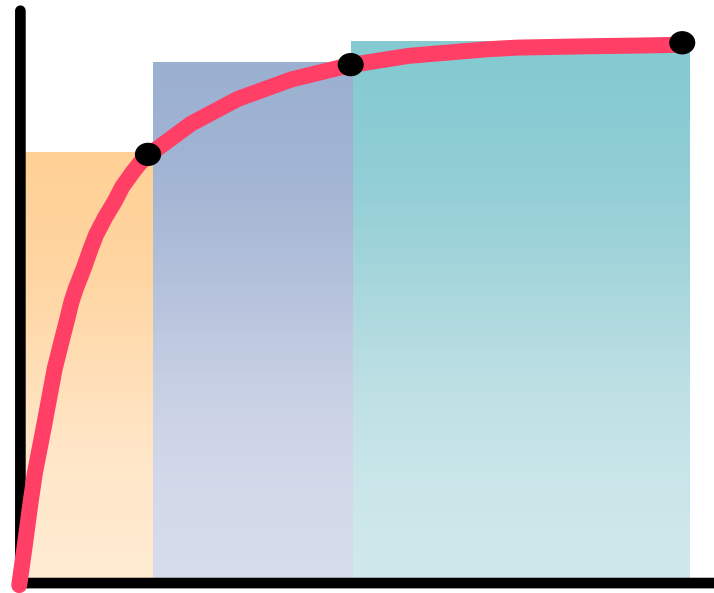


Inventory Management and more

PART IV



FORECASTS

Demand management

- forecasting
- Order processing

Characteristics

- Demand patterns to be identified
- Trend
- Seasonality
- Random variation
- Stable versus dynamic
- Dependent versus independent demand

FORECASTS

Collection and preparation of data

Forecasting techniques

- Qualitative...on judgment
- Extrinsic : external factors
- Intrinsic
 - Average demand if quite steady
 - Moving averages with little seasonality
 - Exponential smoothing : the new data can be given any weight wanted

FORECASTS

- Seasonality
 - Index
 - Forecasts and annual demand average
 - Deseasonalized demand
 - Forecast error
 - Mean absolute deviation
 - Normal distribution
 - Production lead time / Demand lead time ratio

Patterns of Demand

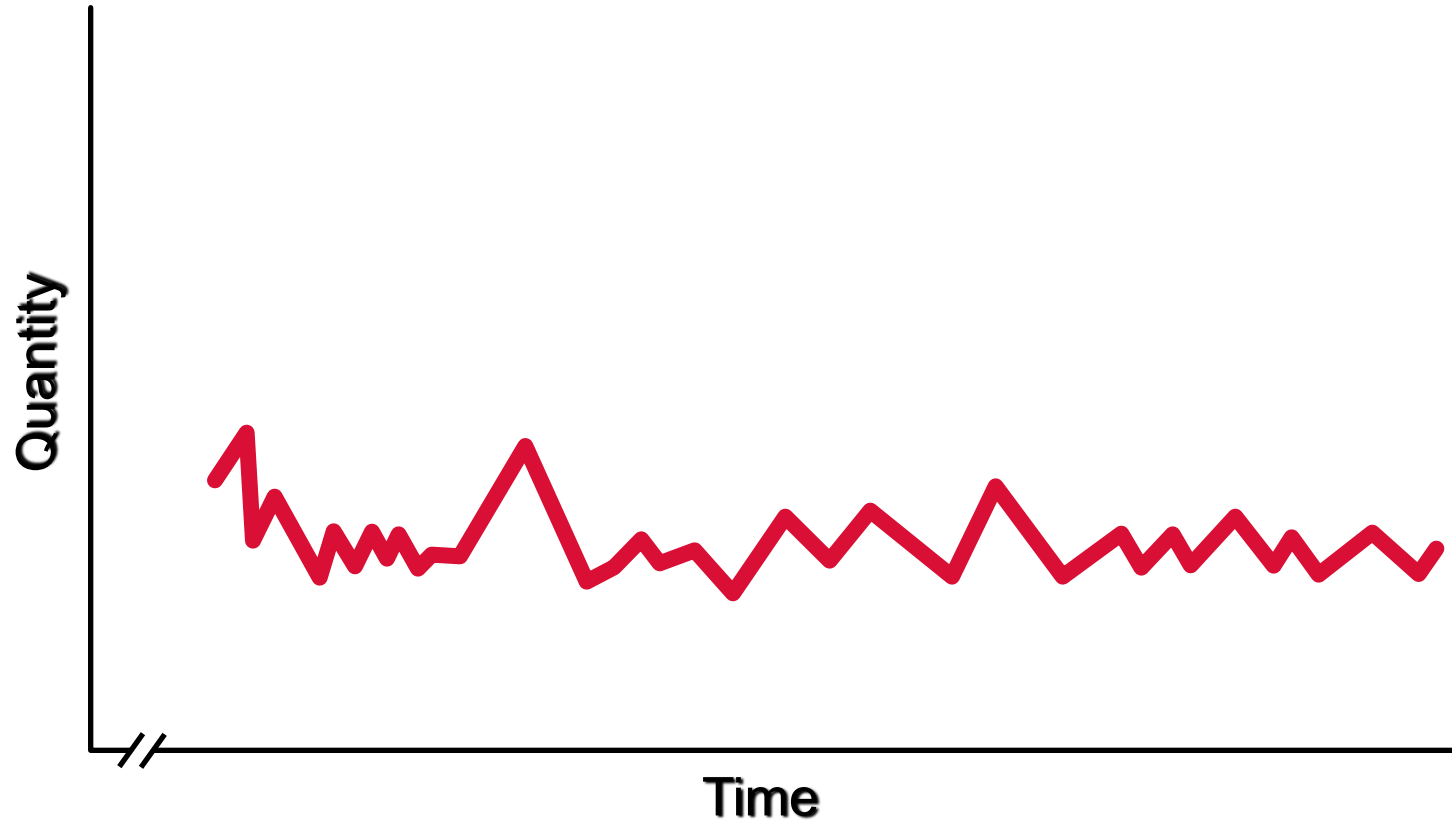
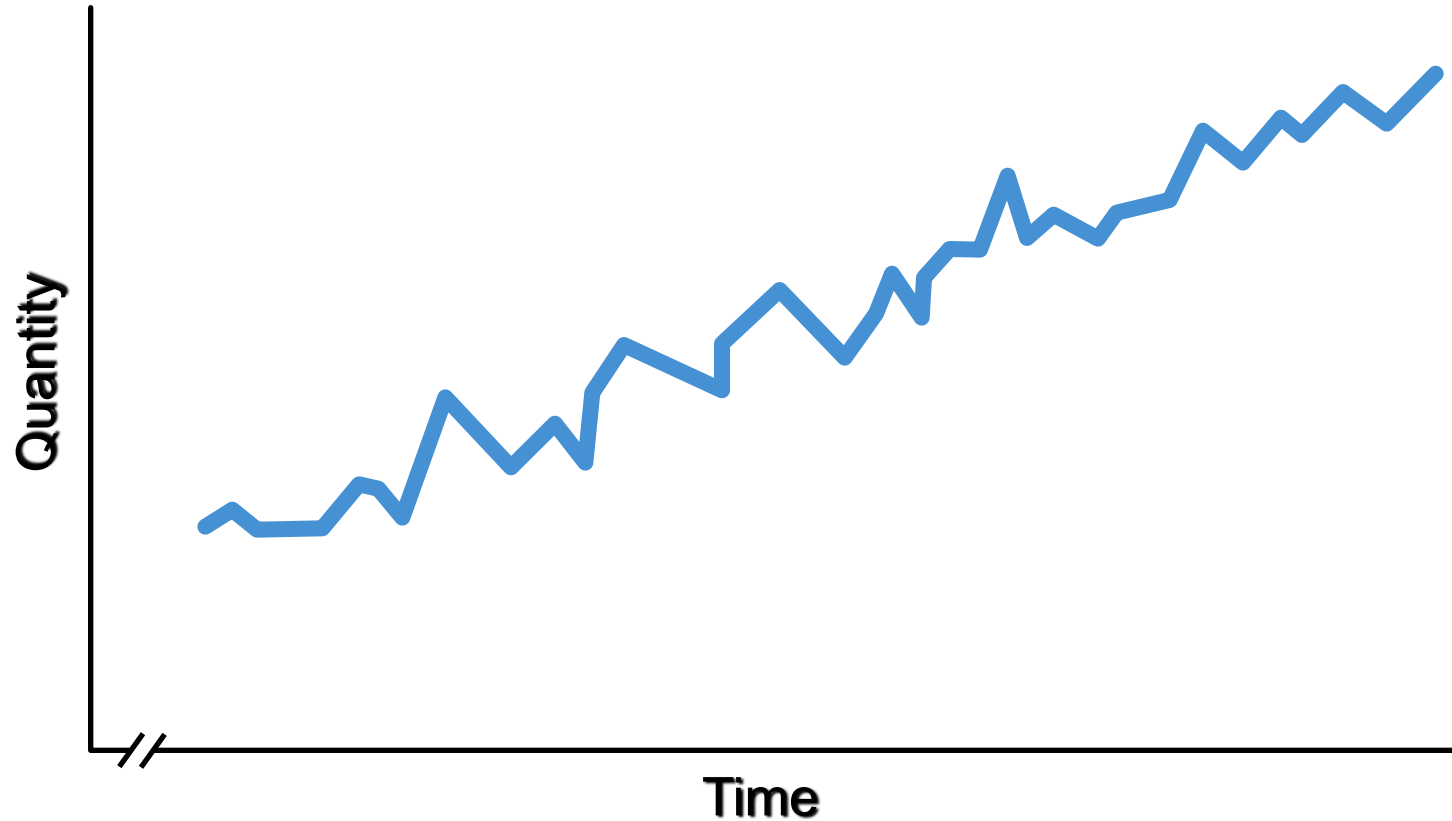


Figure 13.1

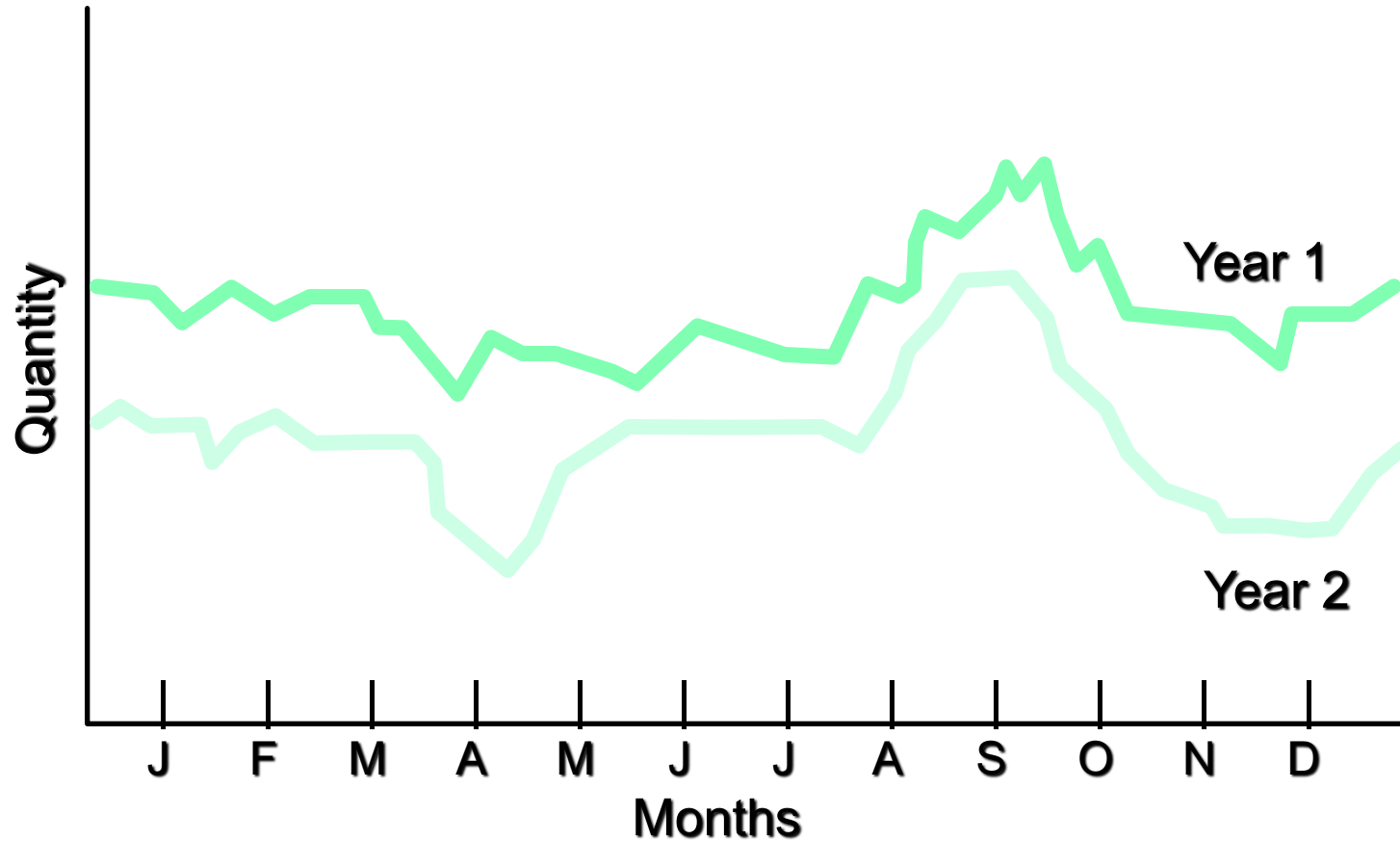
(a) Horizontal: Data cluster about a horizontal line.

Patterns of Demand



(b) Trend: Data consistently increase or decrease.

Patterns of Demand



(c) Seasonal: Data consistently show peaks and valleys.

FUTURE TIME HORIZON

- **Short-range forecast**

- This forecast has a time span of **up to 1 year** but is generally less than 3 months. It is used for planning purchasing, job scheduling, workforce levels, job assignments, and production levels.

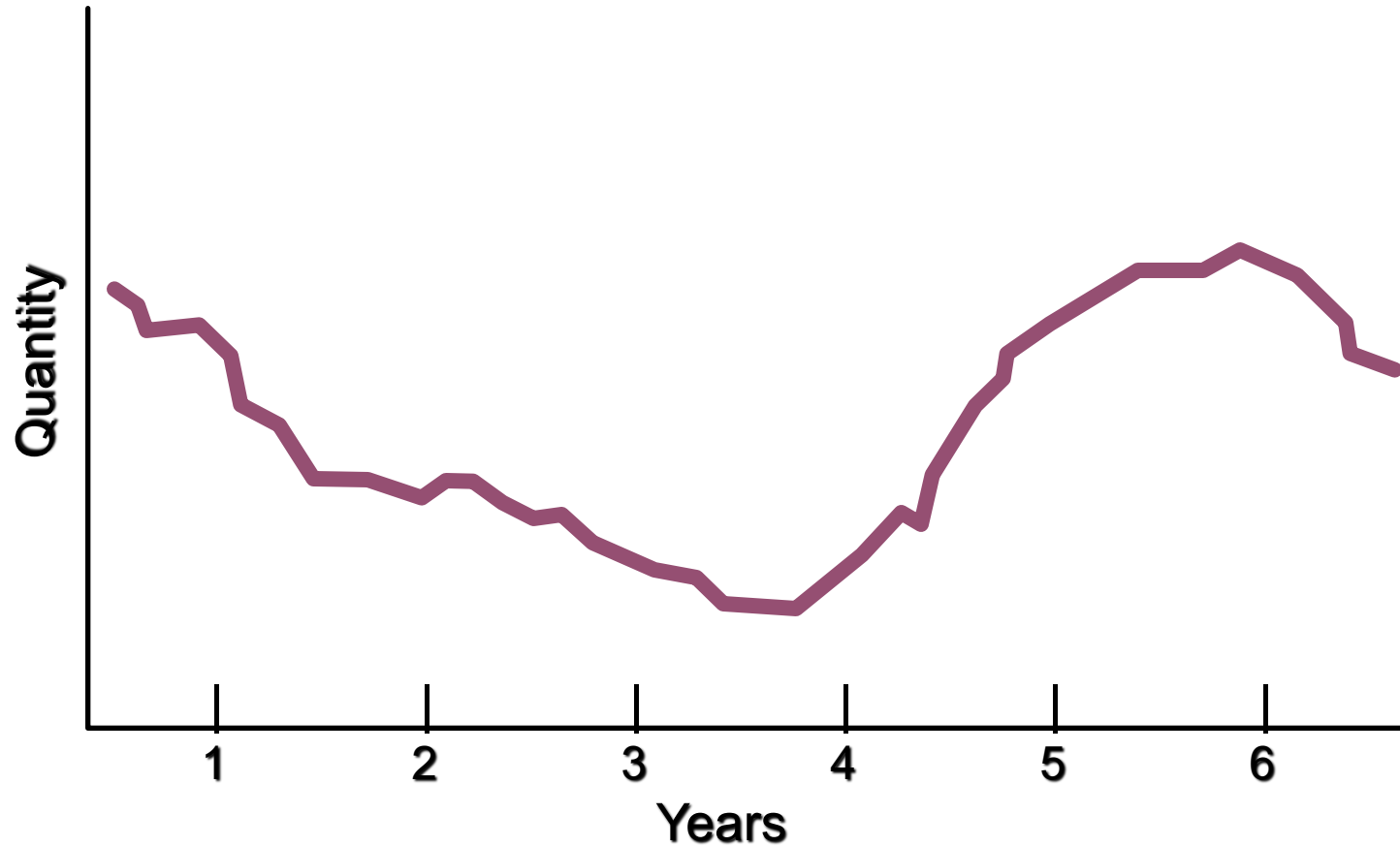
- **Medium-range forecast**

- A medium-range, or intermediate, forecast generally spans from 3 months to 3 years. It is useful in sales **planning, production planning and budgeting**, cash budgeting, and analysis of various operating plans.

- **Long-range forecast**

- Generally 3 years or more in time span, long-range forecasts are used in planning for **new products, capital expenditures, facility location** or expansion, and research and development.

Patterns of Demand



(d) Cyclical: Data reveal gradual increases and decreases over extended periods.

Demand forecast application

Economic forecasts

- *Planning indicators that are valuable in helping organizations prepare medium-to long-range forecasts.*

Technological forecasts

- *Long-term forecasts concerned with the rates of technological progress.*

Demand forecasts

- *Projections of a company's sales for each time period in the planning horizon.*

Connection with techniques

TABLE 13.1 DEMAND FORECAST APPLICATIONS

Application	Time Horizon		
	Short Term (0–3 months)	Medium Term (3 months– 2 years)	Long Term (more than 2 years)
Forecast quantity	Individual products or services	Total sales Groups or families of products or services	Total sales
Decision area	Inventory management Final assembly scheduling Workforce scheduling Master production scheduling	Staff planning Production planning Master production scheduling Purchasing Distribution	Facility location Capacity planning Process management
Forecasting technique	Time series Causal Judgment	Causal Judgment	Causal Judgment

Qualitative method

Jury of executive opinion

- *A forecasting technique that uses the opinion of a small group of high-level managers to form a group estimate of demand.*

Delphi method

- *A forecasting technique using a group process that allows experts to make forecasts.*

Sales force composite

- *A forecasting technique based on salespersons' estimates of expected sales.*

Market survey

- *A forecasting method that solicits input from customers or potential customers regarding future purchasing plans.*

FORECASTS

- **Principles**

- Forecasts are usually wrong
- Every forecast should include an estimate of error
- Forecasts are more accurate **for families or groups**
- Forecasts are more accurate for **nearer time periods**

FORECASTS

Collection and preparation of data

Forecasting techniques

- Qualitative...on judgment
- Extrinsic : external factors
- Intrinsic
 - **Average demand** if quite steady
 - **Moving averages** with little seasonality
 - **Exponential smoothing** : the new data can be given any weight wanted



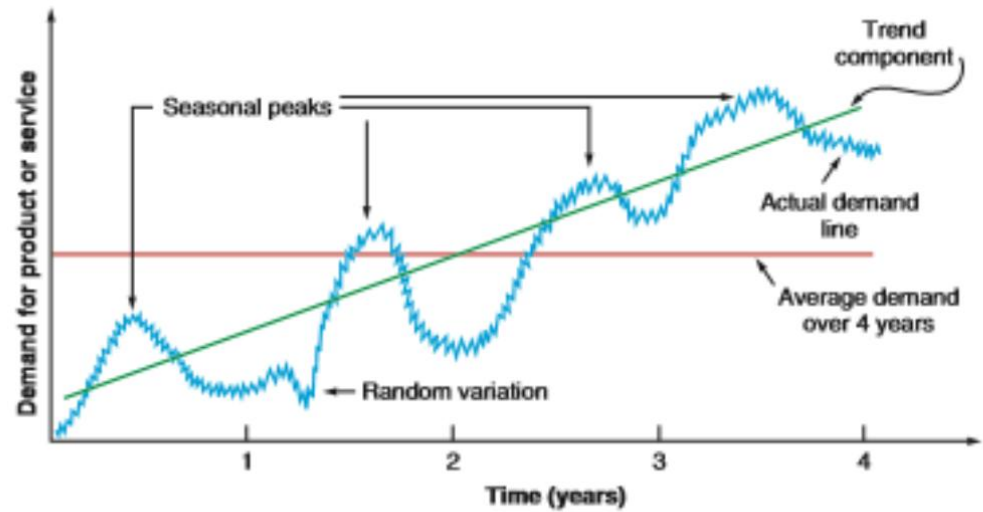
Sum up

ABOUT FORECASTS
TRENDS

ABOUT TIME

WHAT KIND OF
MEASURES' APPROACH

Naive approach



Moving averages

- A forecasting method that uses an average of the n most recent periods of data to forecast the next period.

Donna's Garden Supply wants a 3-month moving-average forecast, including a forecast for next January, for shed sales.

APPROACH ► Storage shed sales are shown in the middle column of the following table. A 3-month moving average appears on the right.

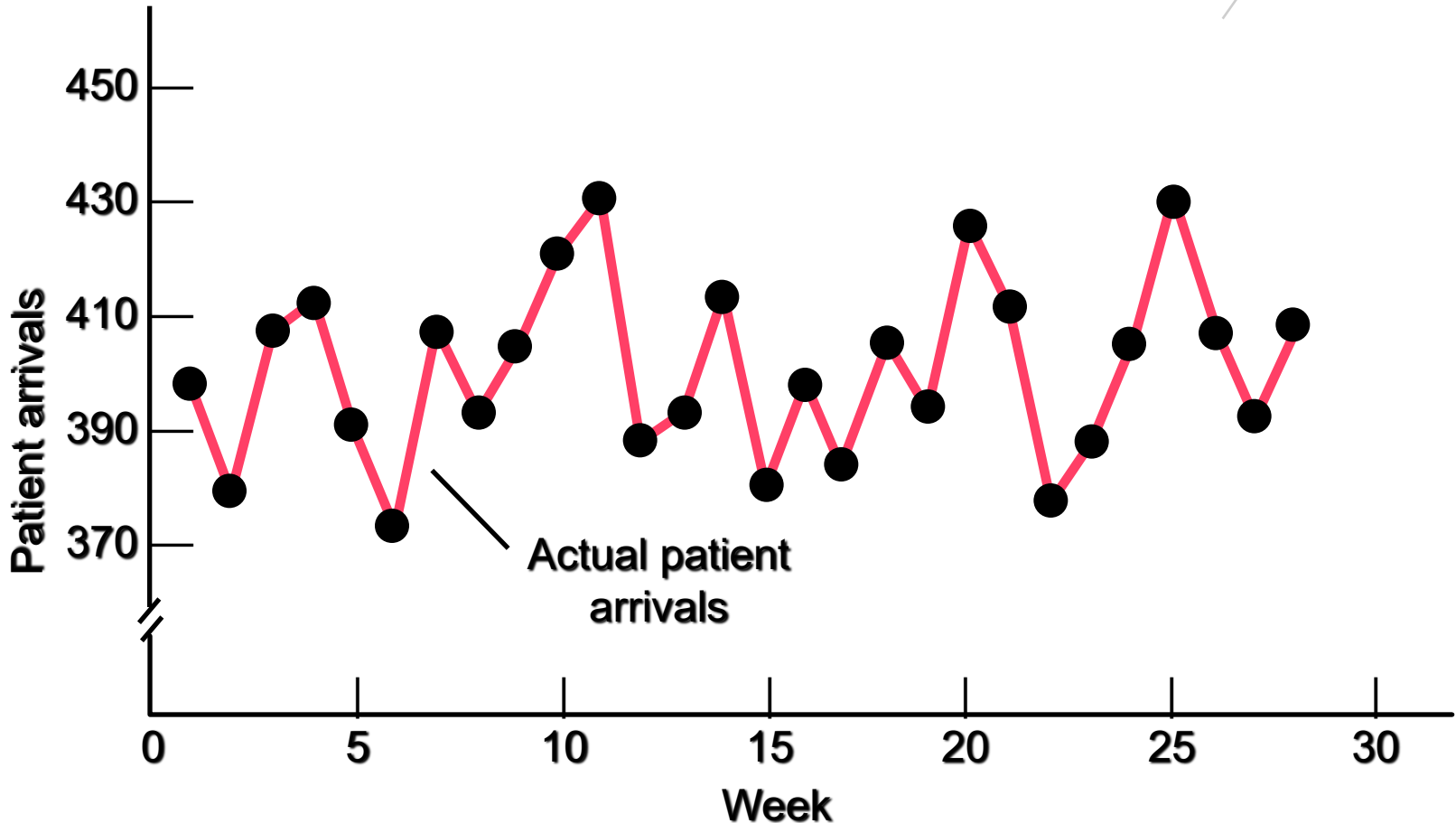
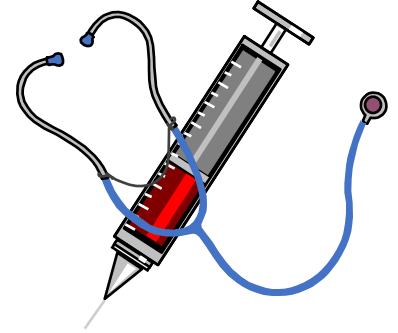
MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\frac{2}{3}$
May	19	$(12 + 13 + 16)/3 = 13\frac{2}{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\frac{1}{3}$
August	30	$(19 + 23 + 26)/3 = 22\frac{2}{3}$
September	28	$(23 + 26 + 30)/3 = 26\frac{1}{3}$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\frac{1}{3}$
December	14	$(28 + 18 + 16)/3 = 20\frac{2}{3}$

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n}$$

where n is the number of periods in the moving average—for example, 4, 5, or 6 months respectively, for a 4-, 5-, or 6-period moving average.

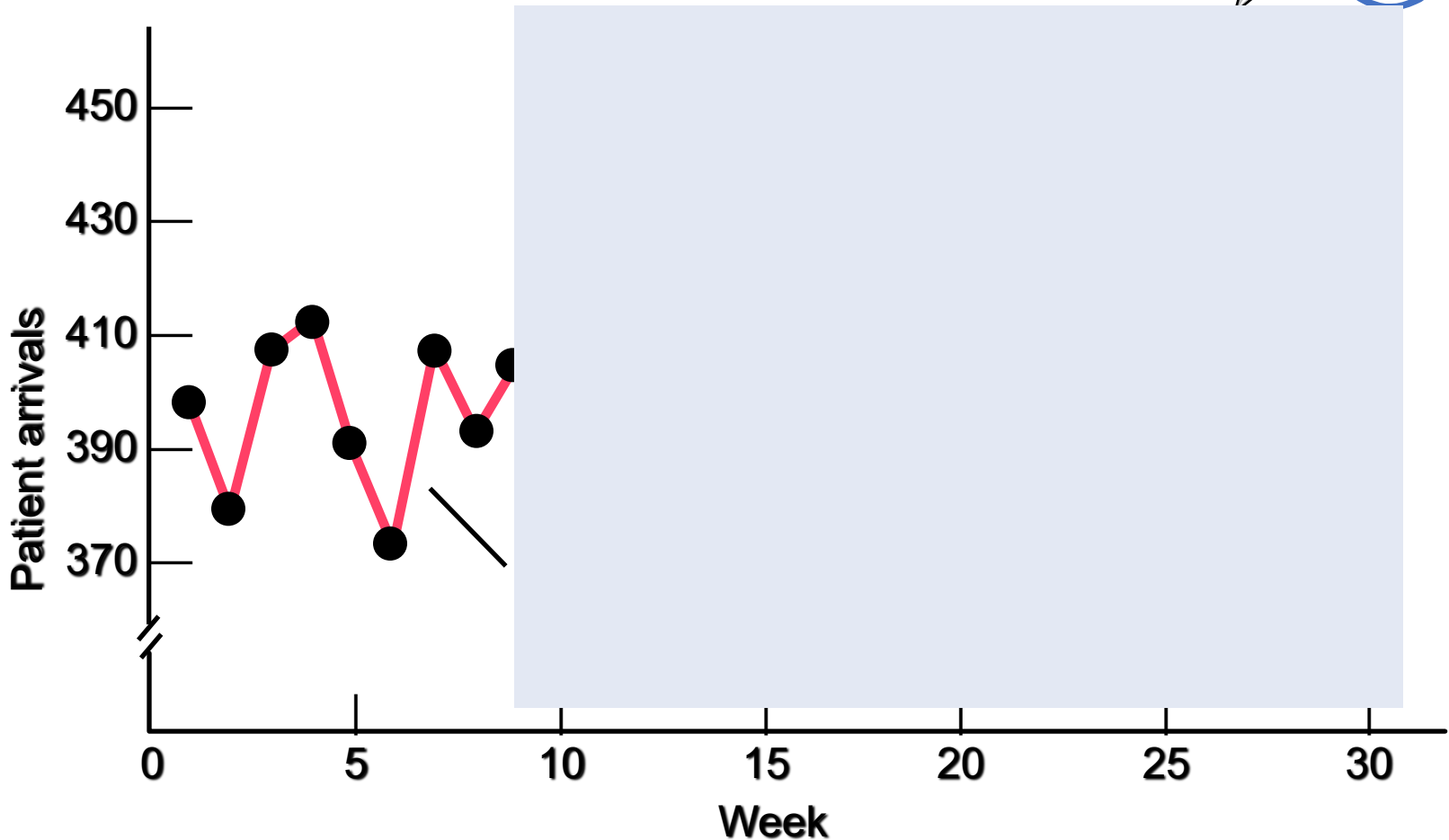
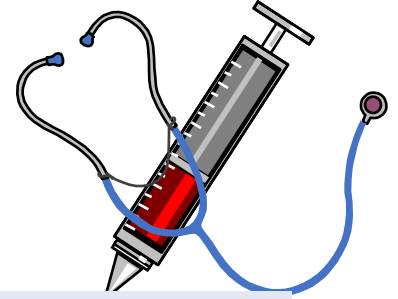
Time-Series Methods

Simple Moving Averages



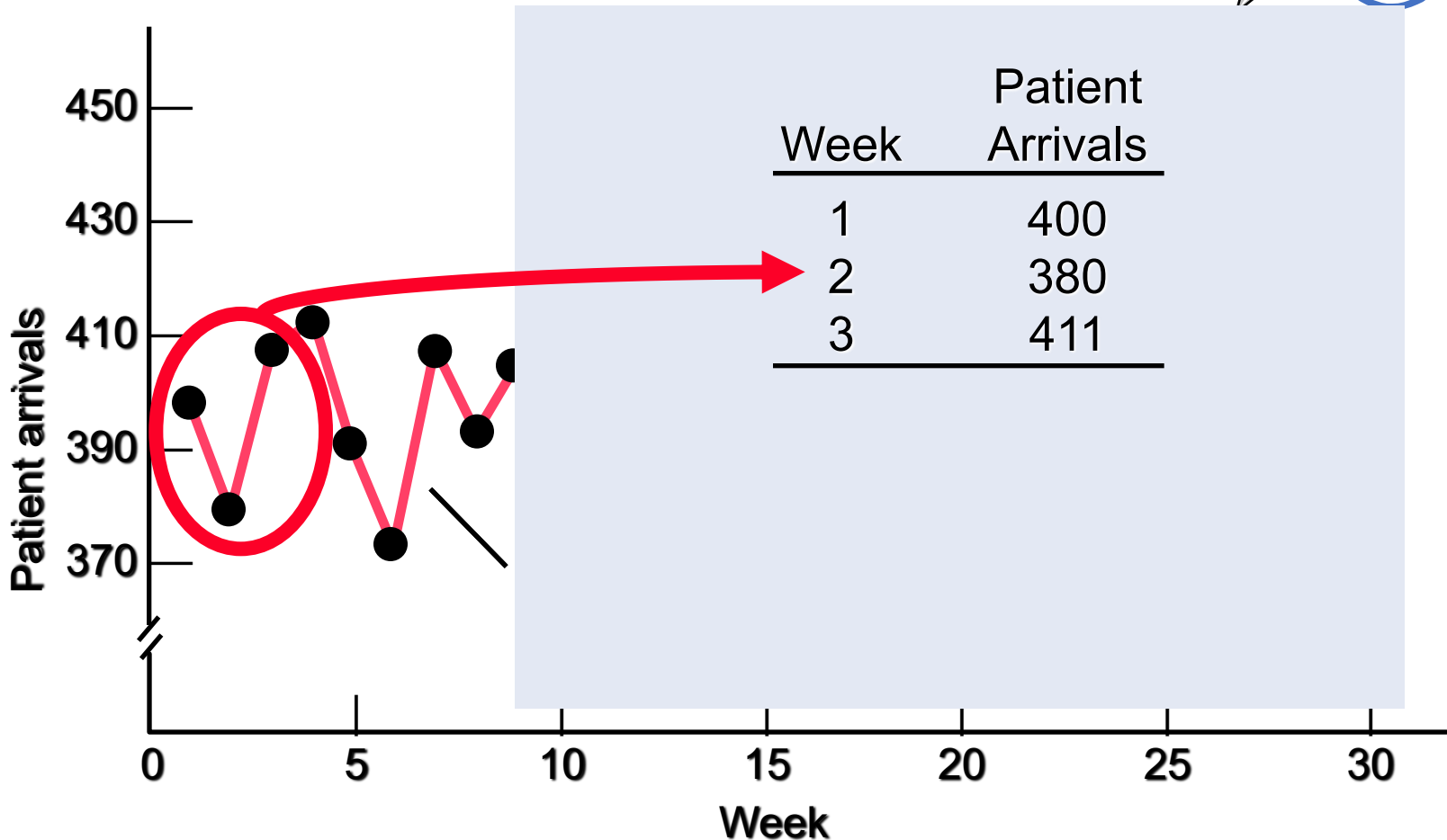
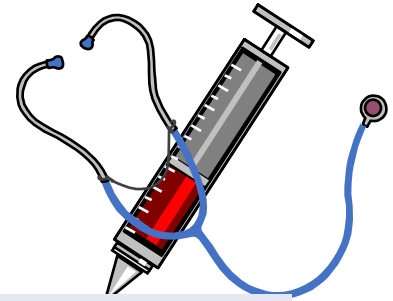
Time-Series Methods

Simple Moving Averages



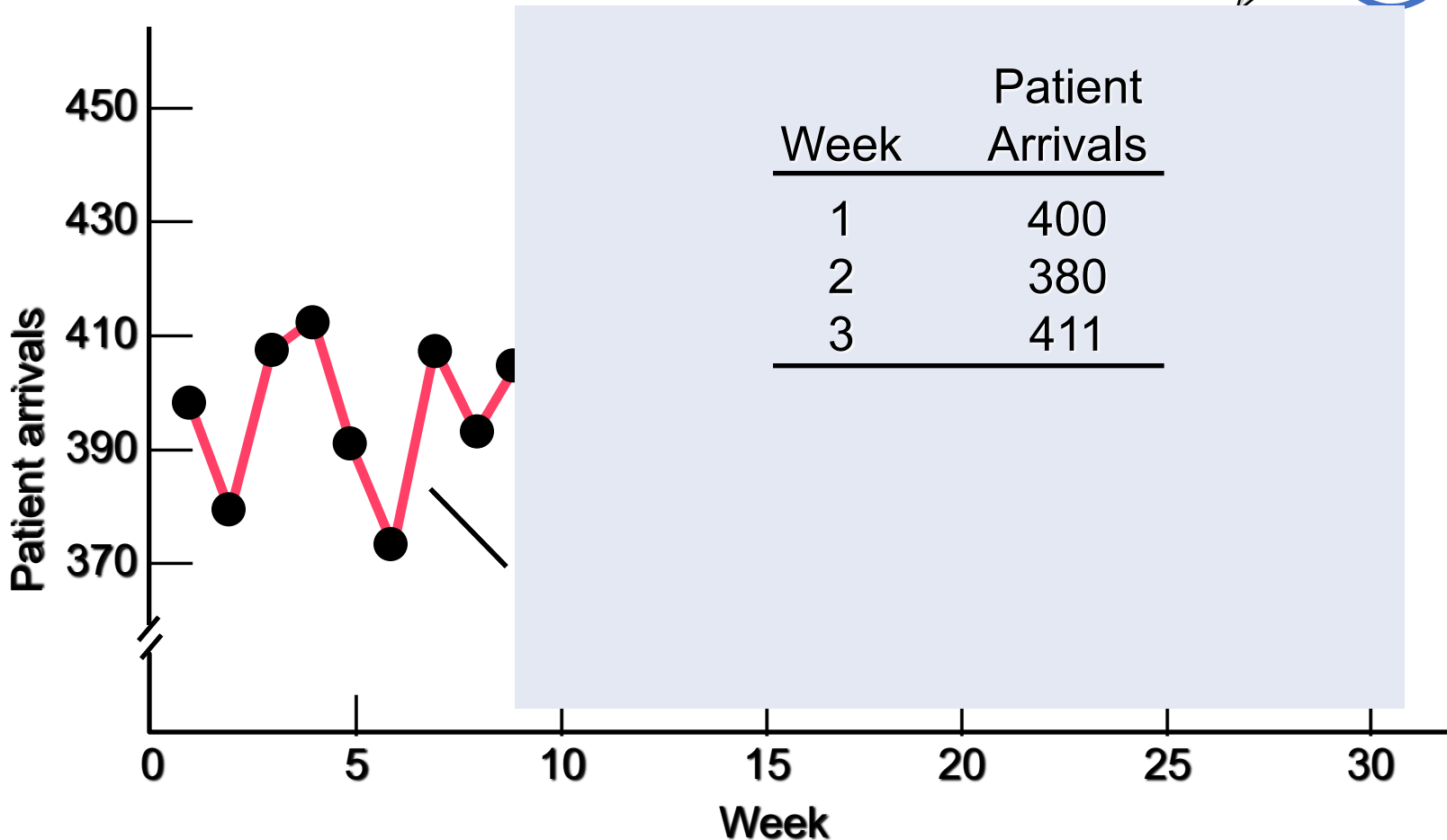
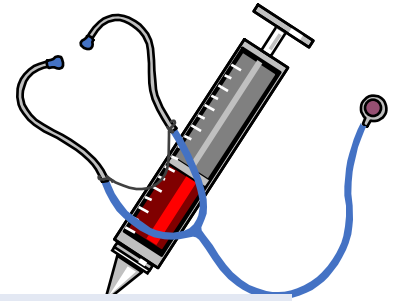
Time-Series Methods

Simple Moving Averages



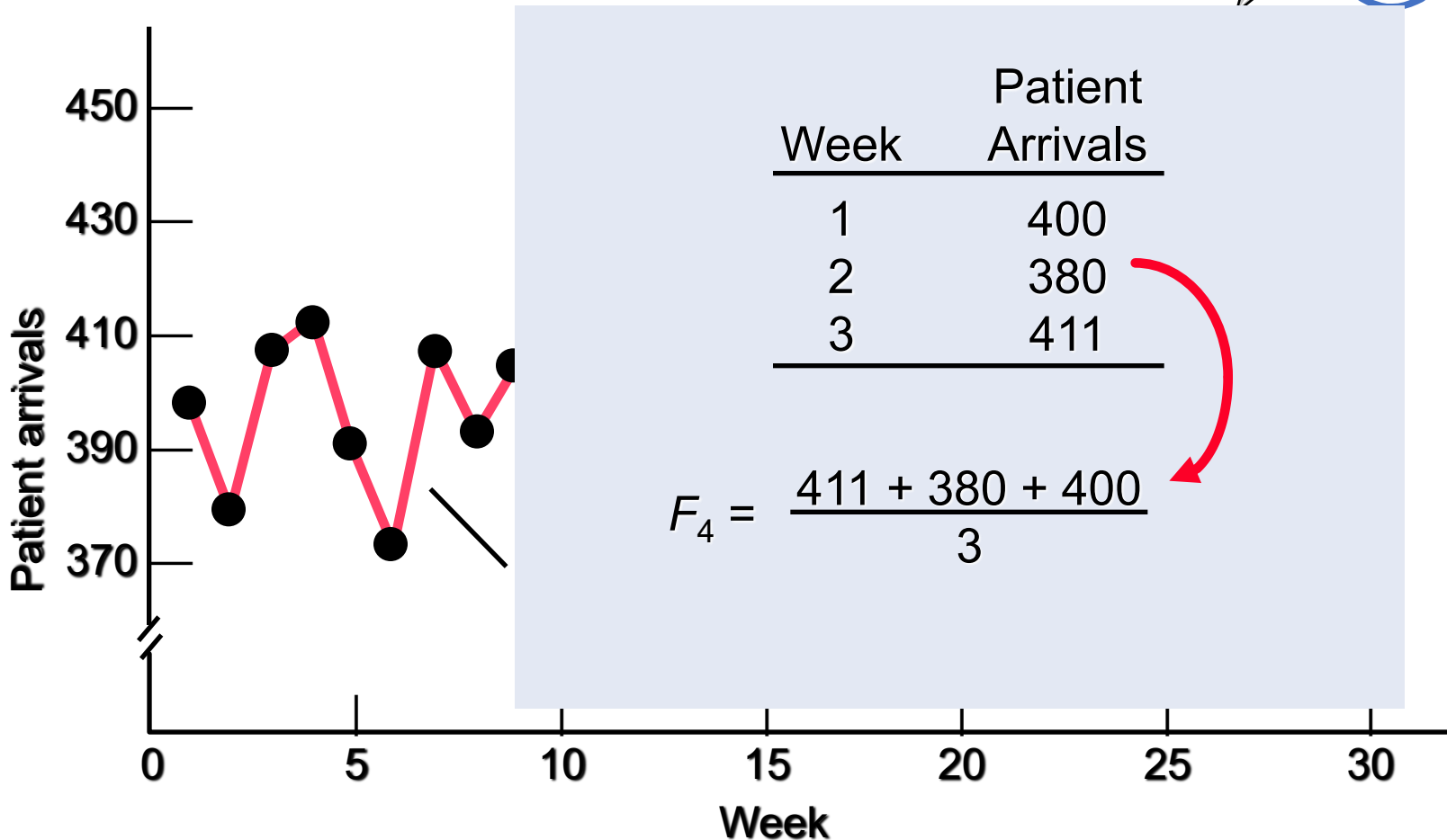
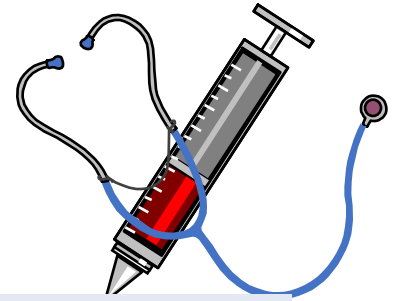
Time-Series Methods

Simple Moving Averages



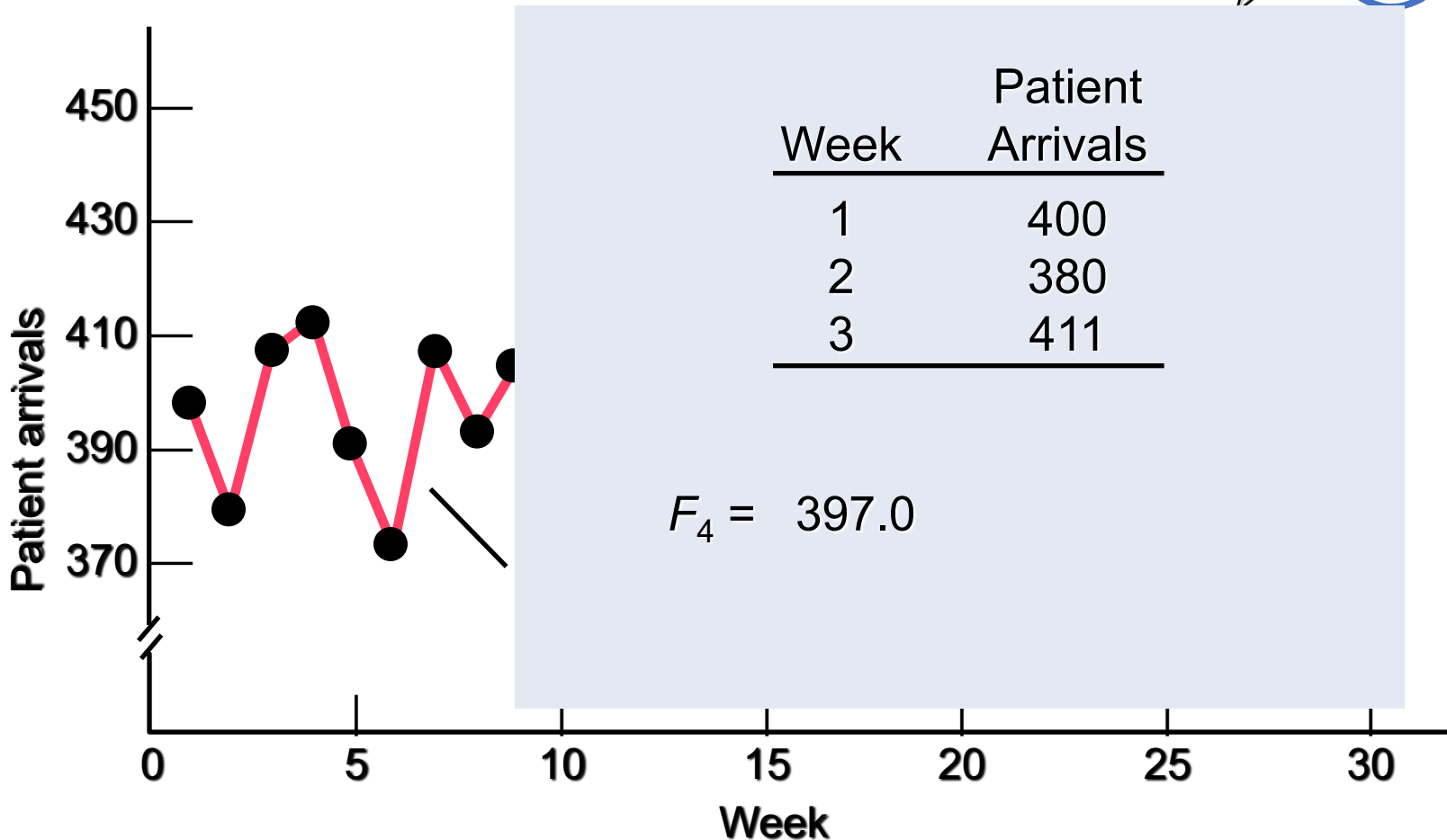
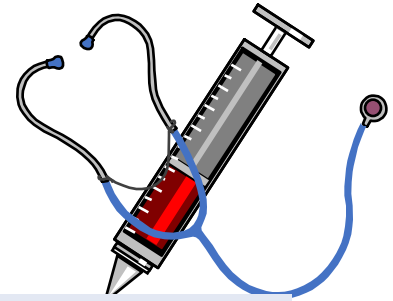
Time-Series Methods

Simple Moving Averages



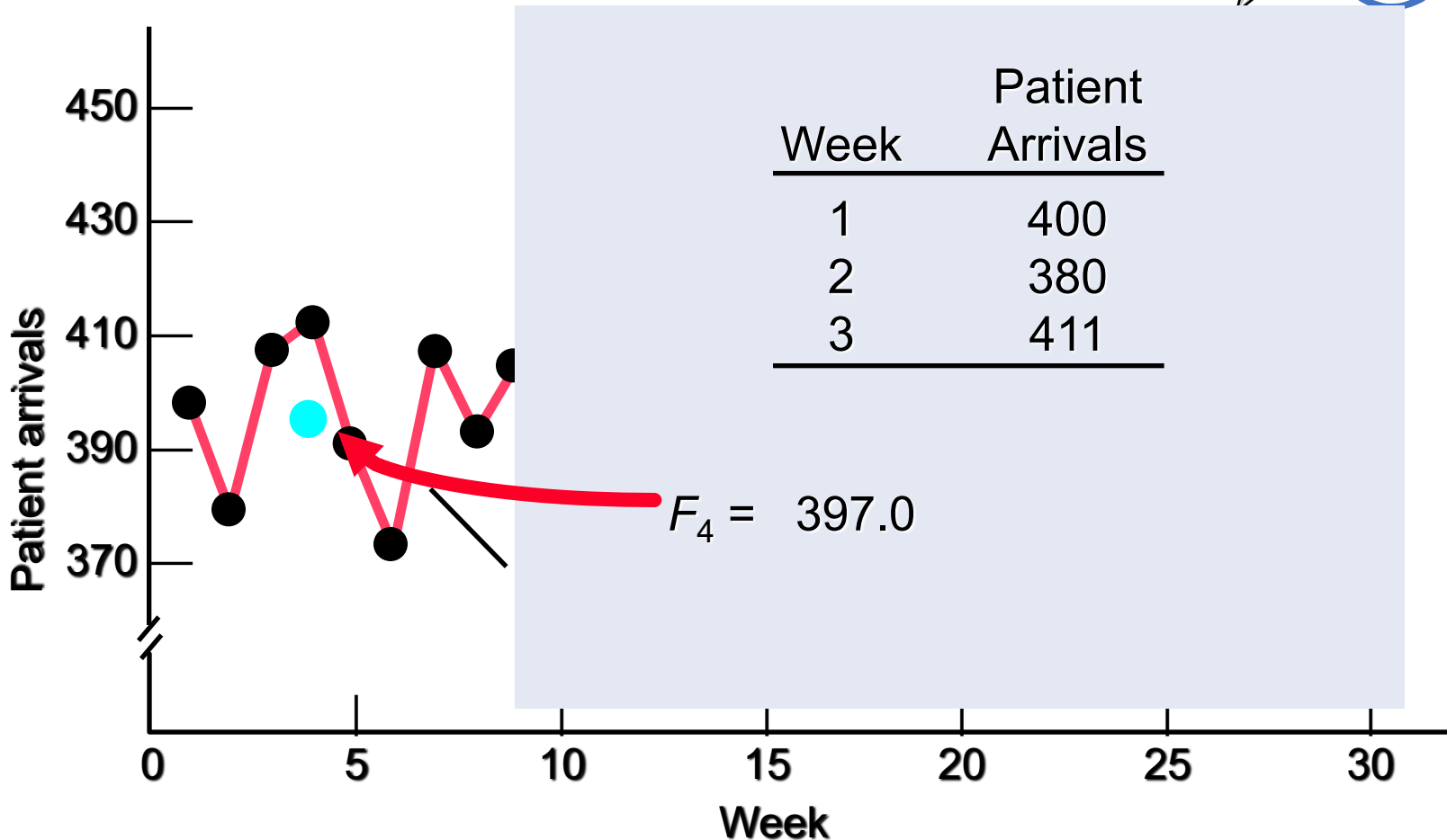
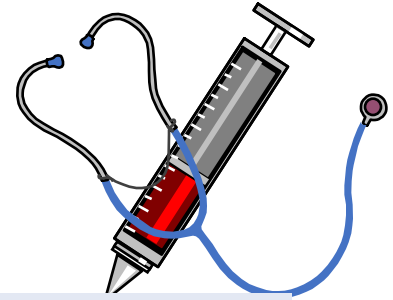
Time-Series Methods

Simple Moving Averages



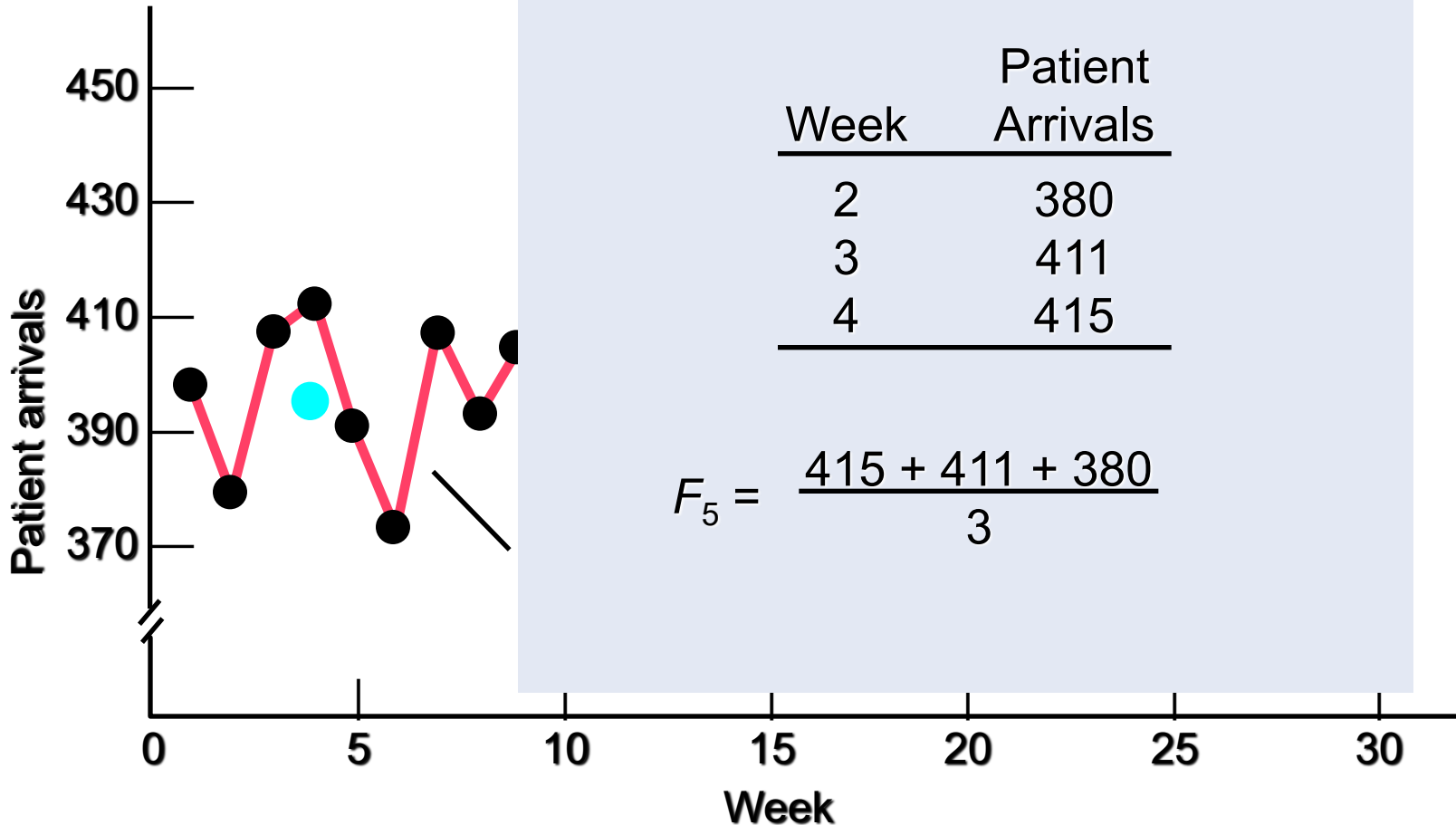
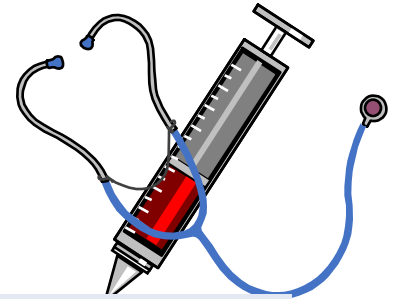
Time-Series Methods

Simple Moving Averages



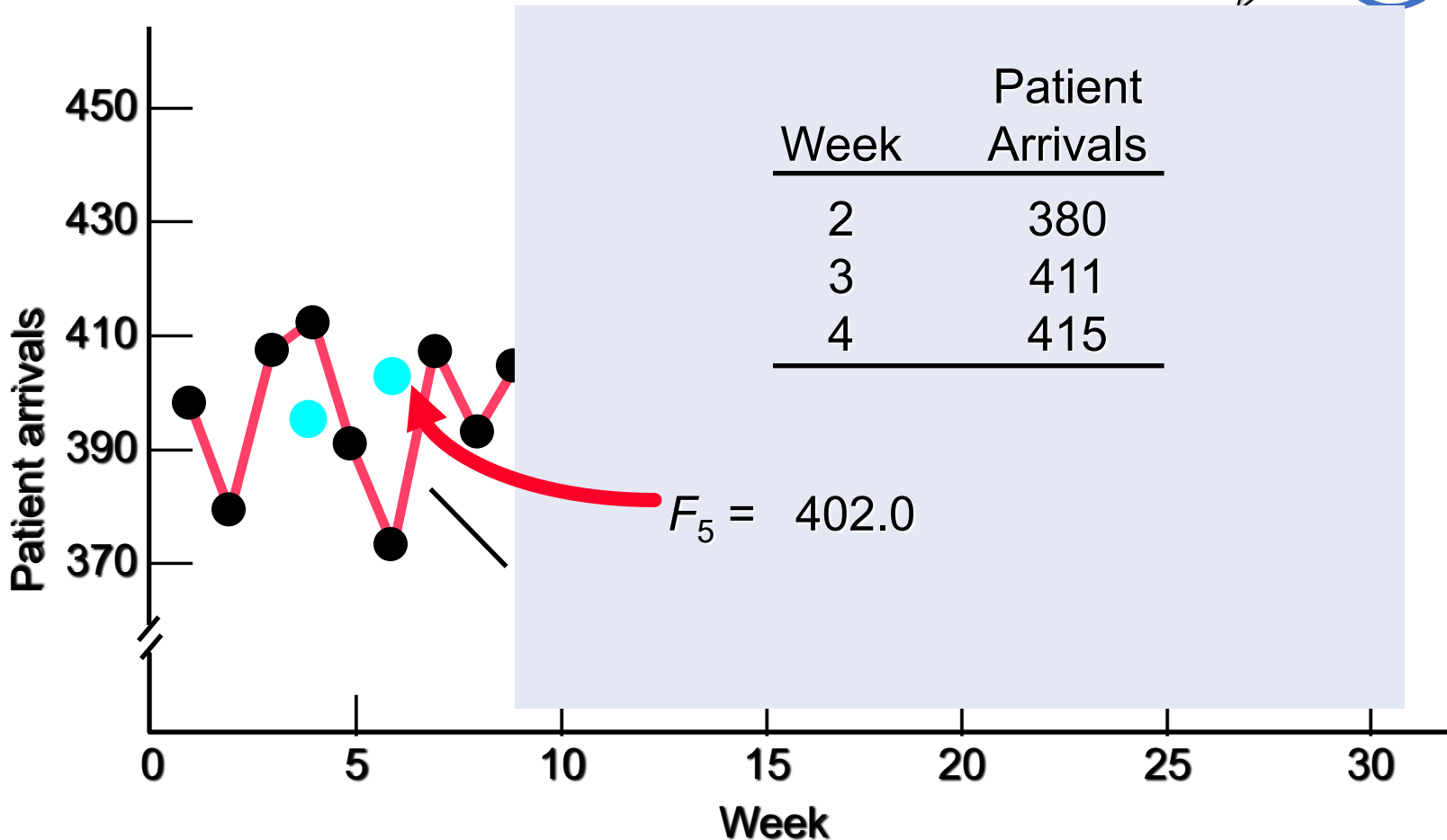
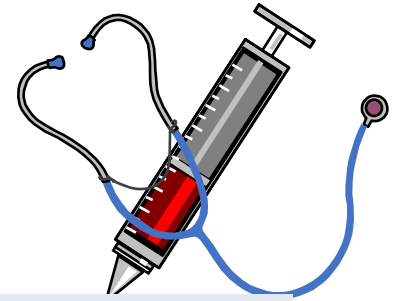
Time-Series Methods

Simple Moving Averages



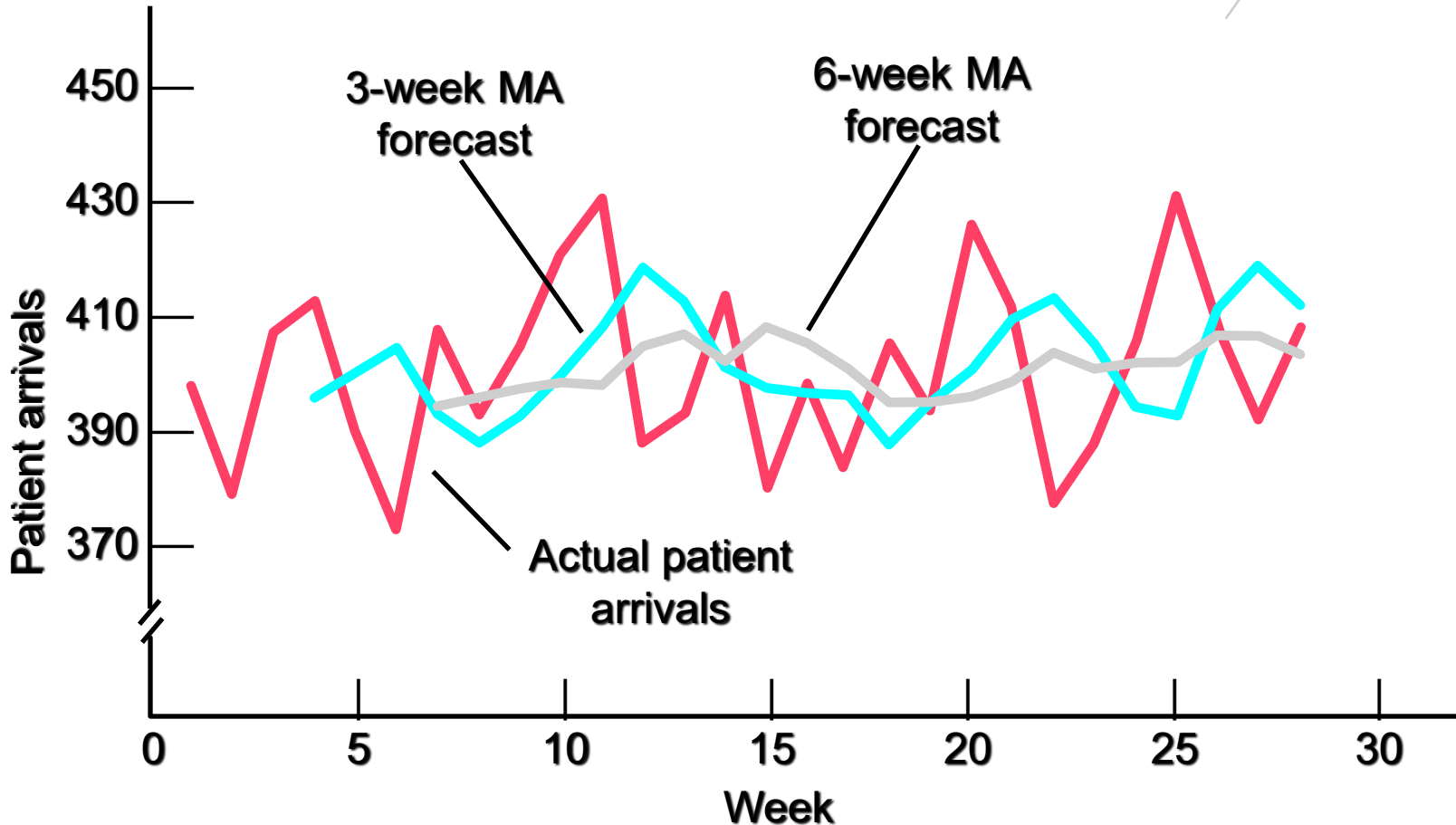
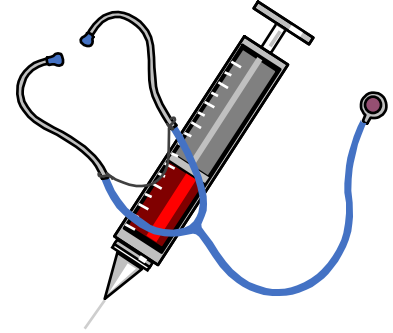
Time-Series Methods

Simple Moving Averages



Time-Series Methods

Simple Moving Averages



DETERMINING THE WEIGHTED MOVING AVERAGE

Donna's Garden Supply (see Example 1) wants to forecast storage shed sales by weighting the past 3 months, with more weight given to recent data to make them more significant.

APPROACH ► Assign more weight to recent data, as follows:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
$\frac{6}{6}$	Sum of weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

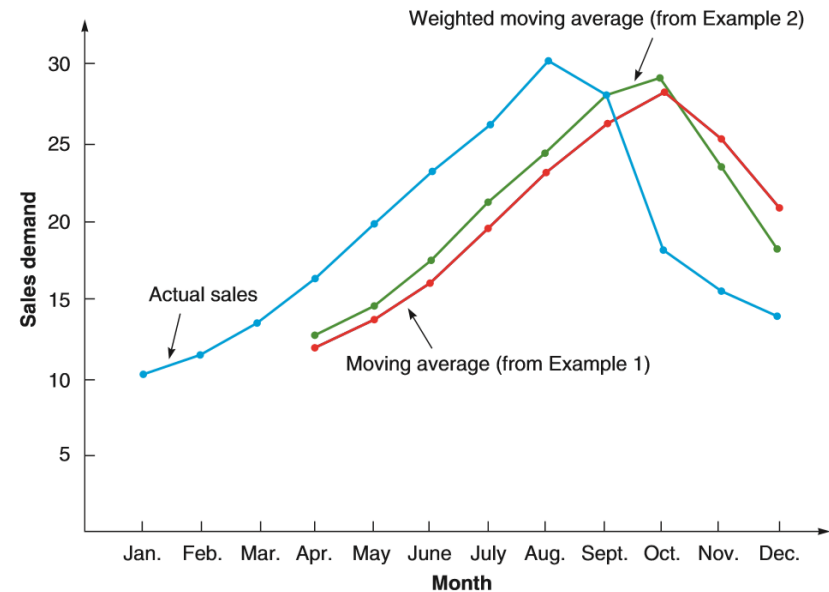
SOLUTION ► The results of this weighted-average forecast are as follows:

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\frac{1}{3}$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\frac{5}{6}$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\frac{1}{2}$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\frac{1}{3}$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\frac{1}{3}$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\frac{2}{3}$

WEIGHTED MOVING AVERAGE

Comparison

- Moving-average methods always lag behind when there is a trend present, as shown by the blue line (actual sales) for January through August.



Exponential Smoothing

Exponential smoothing is another weighted-moving-average forecasting method. It involves very *little* record keeping of past data and is fairly easy to use. The basic exponential smoothing formula can be shown as follows:

$$\begin{aligned} \text{New forecast} = & \text{Last period's forecast} \\ & + \alpha (\text{Last period's actual demand} - \text{Last period's forecast}) \end{aligned} \quad (4-3)$$

where α is a weight, or **smoothing constant**, chosen by the forecaster, that has a value greater than or equal to 0 and less than or equal to 1. Equation (4-3) can also be written mathematically as:

$$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1}) \quad (4-4)$$

where

- F_t = new forecast
- F_{t-1} = previous period's forecast
- α = smoothing (or weighting) constant ($0 \leq \alpha \leq 1$)
- A_{t-1} = previous period's actual demand

Exponential Smoothing

Weighted exponential smoothing

It can be changed to give more weight to recent data (when α is high) or more weight to past data (when α is low).

WEIGHT ASSIGNED TO					
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4TH MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5TH MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$.1	.09	.081	.073	.066
$\alpha = .5$.5	.25	.125	.063	.031

- Better trend identification
- Demand and forecasts are smoothed

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\begin{aligned} \text{Forecast including trend (FIT}_t) = & \text{Exponentially smoothed forecast average (F}_t) \\ & + \text{Exponentially smoothed trend (T}_t) \end{aligned} \quad (4-8)$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \quad (4-9)$$

$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$

or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4-10)$$

where F_t = exponentially smoothed forecast average of the data series in period t

T_t = exponentially smoothed trend in period t

A_t = actual demand in period t

α = smoothing constant for the average ($0 \leq \alpha \leq 1$)

β = smoothing constant for the trend ($0 \leq \beta \leq 1$)

Changing trend

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1-5
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\text{Forecast including trend (FIT)} = \text{Exponentially smoothed forecast average (F}_t\text{)} + \text{Exponentially smoothed trend (T}_t\text{)} \quad (4-8)$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \quad (4-9)$$

$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$

or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4-10)$$

where F_t = exponentially smoothed forecast average of the data series in period t

T_t = exponentially smoothed trend in period t

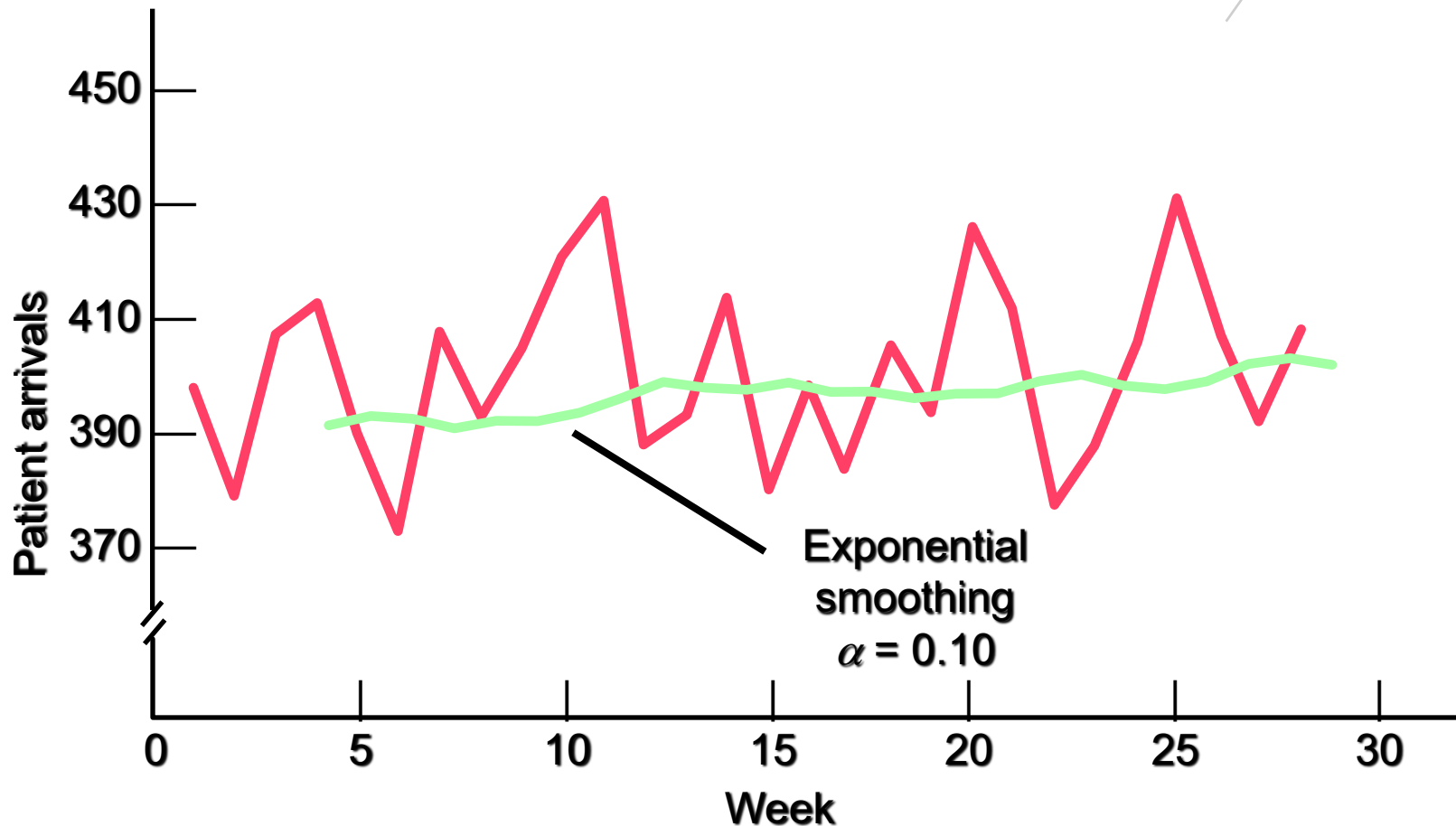
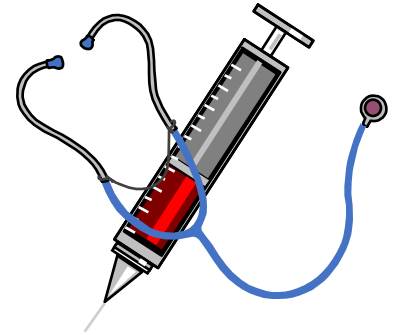
A_t = actual demand in period t

α = smoothing constant for the average ($0 \leq \alpha \leq 1$)

β = smoothing constant for the trend ($0 \leq \beta \leq 1$)

Time-Series Methods

Exponential Smoothing



Exponential Smoothing with Trend Adjustment Example

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

$$\alpha = .2$$

$$\beta = .4$$

Exponential Smoothing with Trend Adjustment Example (1 of 5)

Table 4.2 Forecast with $\alpha = .2$ and $\beta = .4$

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 1: Average for Month 2

$$\begin{aligned} F_2 &= \alpha A_1 + (1 - \alpha)(F_1 + T_1) \\ F_2 &= (.2)(12) + (1 - .2)(11 + 2) \\ &= 2.4 + (.8)(13) = 2.4 + 10.4 \\ &= 12.8 \text{ units} \end{aligned}$$

Exponential Smoothing with Trend Adjustment Example (2 of 5)

Table 4.2 Forecast with $\alpha = .2$ and $\beta = .4$

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 2: Trend for Month 2

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$

$$\begin{aligned} T_2 &= (.4)(12.8 - 11) + (1 - .4)(2) \\ &= .72 + 1.2 = 1.92 \text{ units} \end{aligned}$$

Exponential Smoothing with Trend Adjustment Example (3 of 5)

Table 4.2 Forecast with $\alpha = .2$ and $\beta = .4$

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 3: Calculate FIT for Month 2

$$FIT_2 = F_2 + T_2$$
$$FIT_2 = 12.8 + 1.92$$
$$= 14.72 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example (4 of 5)

Table 4.2 Forecast with $\alpha = .2$ and $\beta = .4$

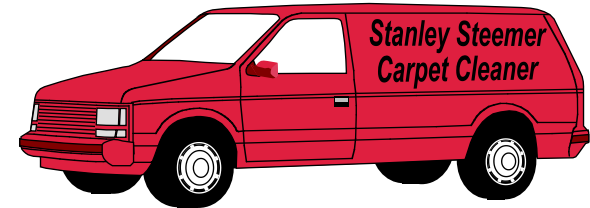
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10		32.48	2.68	35.16

Seasonal indices

1. Find the *average historical demand each season* (or month in this case) by summing the demand for that month in each year and dividing by the number of years of data available. For example, if, in January, we have seen sales of 8, 6, and 10 over the past 3 years, average January demand equals $(8 + 6 + 10)/3 = 8$ units.
2. Compute the *average demand over all months* by dividing the total average annual demand by the number of seasons. For example, if the total average demand for a year is 120 units and there are 12 seasons (each month), the average monthly demand is $120/12 = 10$ units.
3. Compute a *seasonal index* for each season by dividing that *month's* historical average demand (from Step 1) by the average demand over all months (from Step 2). For example, if the average historical January demand over the past 3 years is 8 units and the average demand over all months is 10 units, the seasonal index for January is $8/10 = .80$. Likewise, a seasonal index of 1.20 for February would mean that February's demand is 20% larger than the average demand over all months.
4. Estimate next year's total annual demand.
5. Divide this estimate of total annual demand by the number of seasons, then multiply it by the seasonal index for each month. This provides the *seasonal forecast*.

Time-Series Methods

Seasonal Influences



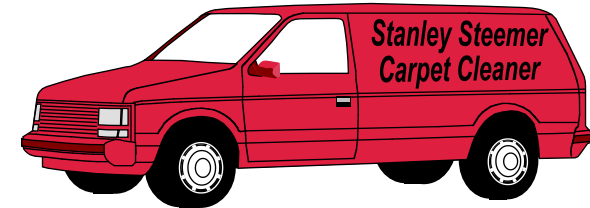
Quarter	Year 1	Year 2	Year 3	Year 4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215
Total	1000	1200	1800	2200

Seasonal variations

Regular upward or downward movements in a time series that tie to recurring events.

Time-Series Methods

Seasonal Influences



Period

Starting Year

Years

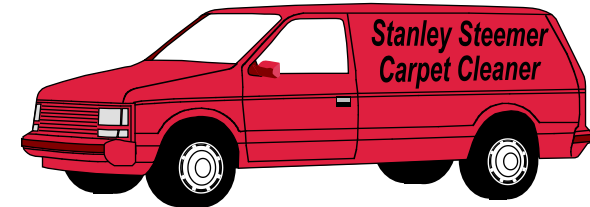
Computed Forecast Demand for Year 5

User-supplied Forecast Demand for Year 5

Quarter	Year			
	1	2	3	4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215

Time-Series Methods

Seasonal Influences



Period

Quarter

Seasonal

Forecast

Starting Year

Quarter

1

0.2043

132.795

2

1.2979

843.635

Computed Forecast

3

2.0001

1300.065

4

0.4977

323.505

User-supplied Forecast

Year

Quarter

1

2

3

4

1

45

70

100

100

2

335

370

585

725

3

520

590

830

1160

4

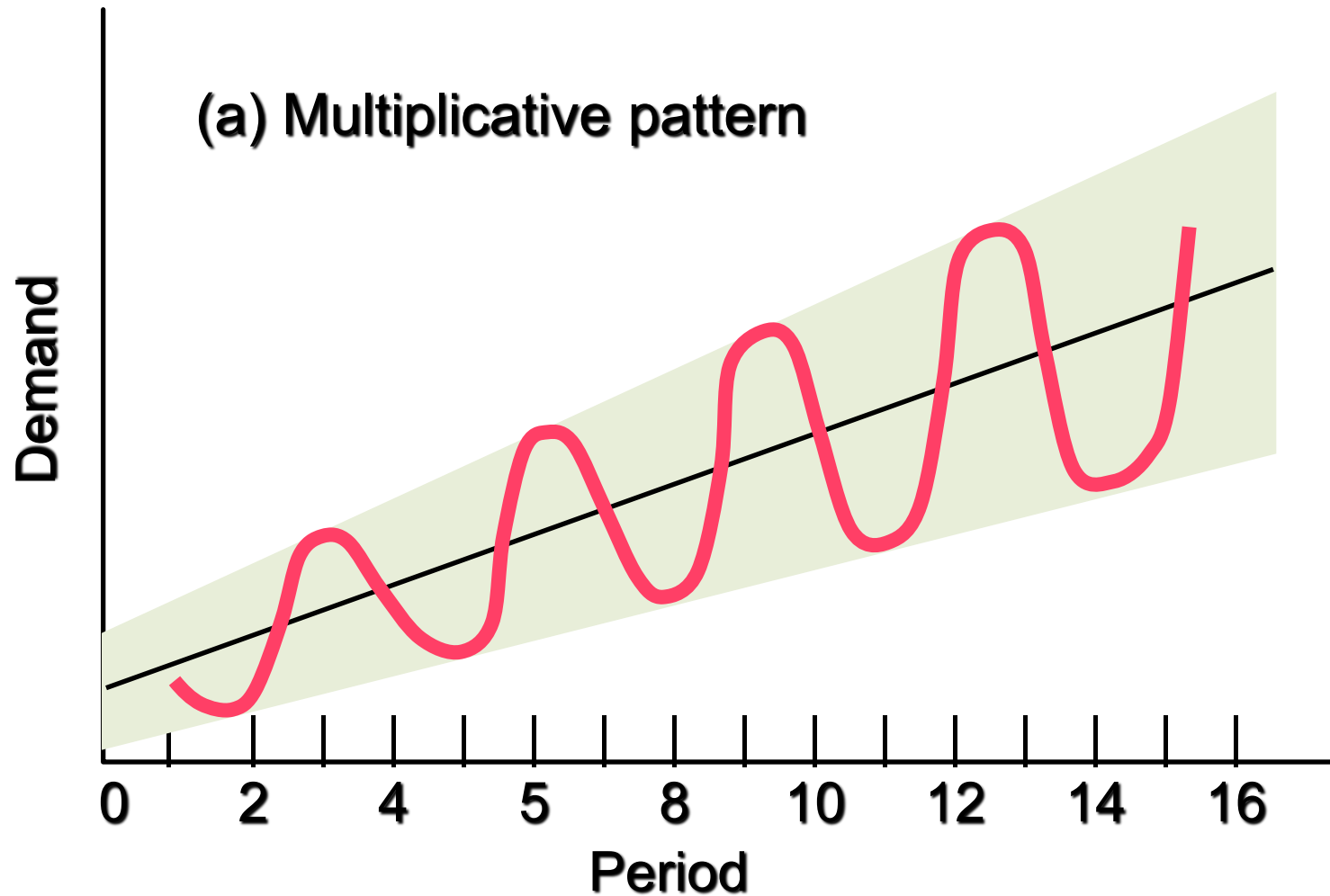
100

170

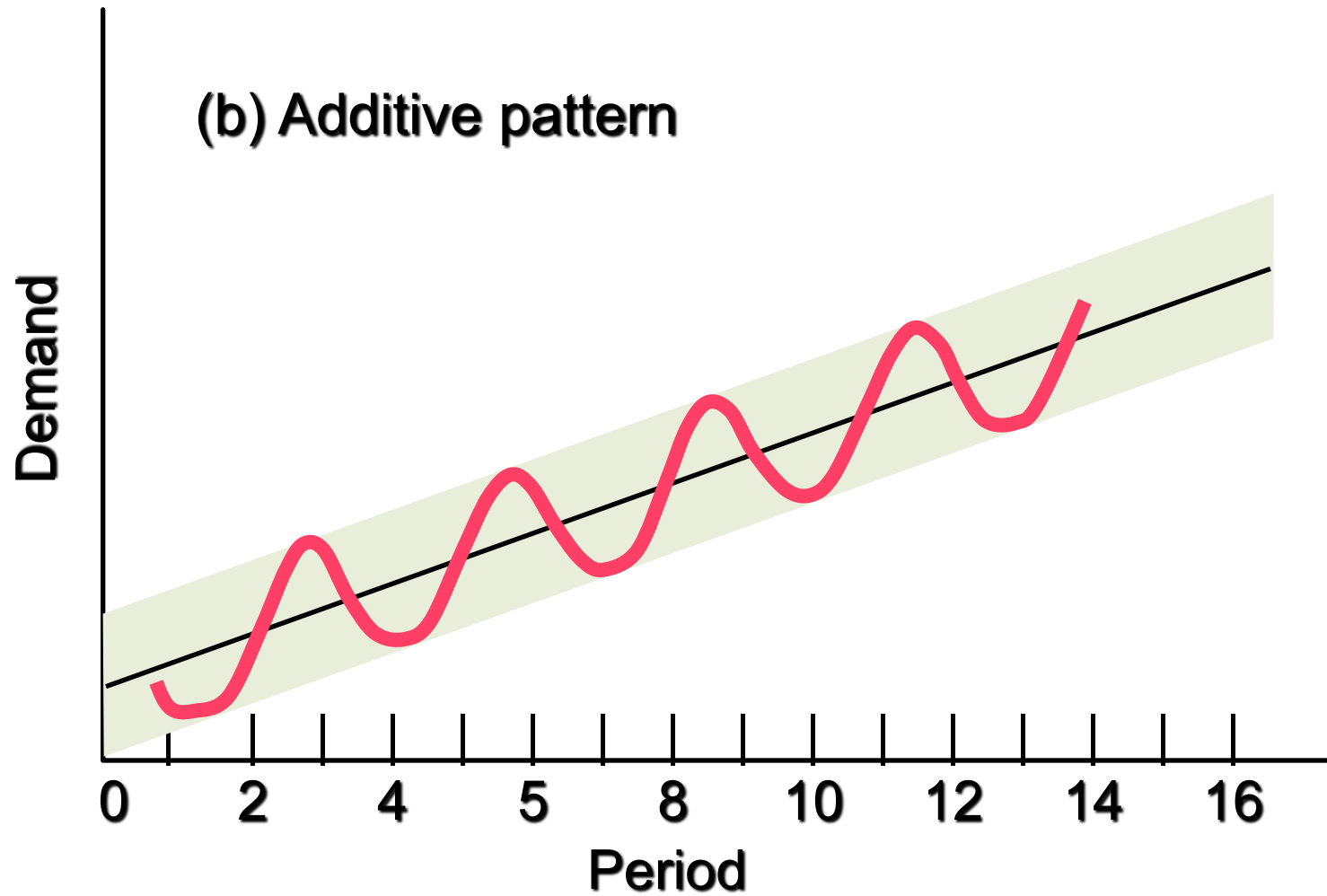
285

215

Seasonal Patterns



Seasonal Patterns



Seasonal Index Example (1 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90		
Feb	70	85	85	80		
Mar	80	93	82	85		
Apr	90	95	115	100		
May	113	125	131	123		
June	110	115	120	115		
July	100	102	113	105		
Aug	88	102	110	100		
Sept	85	90	95	90		
Oct	77	78	85	80		
Nov	75	82	83	80		
Dec	82	78	80	80		
Total average annual demand =				1,128		

Seasonal Index Example (2 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	
Feb	70	85	85	80	94	
Mar	75	90	95	85	94	
Apr	90	100	100	90	94	
May	83	95	95	91	94	
June	95	100	100	95	94	
July	100	102	115	105	94	
Aug	88	102	110	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
Total average annual demand =				1,128		

$$\text{Average monthly demand} = \frac{1,128}{12 \text{ months}} = 94$$

Seasonal Index Example (3 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957 (= 90/94)
Feb	70	85	85	80	94	
Mar	80	93	82	85	94	
Apr	90	95	115	100	94	
$\text{Seasonal index} = \frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}$						
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
Total average annual demand = 1,128						

Seasonal Index Example (4 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957 (= 90/94)
Feb	70	85	85	80	94	.851 (= 80/94)
Mar	80	93	82	85	94	.904 (= 85/94)
Apr	90	95	115	100	94	1.064 (= 100/94)
May	113	125	131	123	94	1.309 (= 123/94)
June	110	115	120	115	94	1.223 (= 115/94)
July	100	102	113	105	94	1.117 (= 105/94)
Aug	88	102	110	100	94	1.064 (= 100/94)
Sept	85	90	95	90	94	.957 (= 90/94)
Oct	77	78	85	80	94	.851 (= 80/94)
Nov	75	82	83	80	94	.851 (= 80/94)
Dec	82	78	80	80	94	.851 (= 80/94)
Total average annual demand =				1,128		

Seasonal Index Example (5 of 6)

Seasonal forecast for Year 4

MONTH	DEMAND	MONTH	DEMAND
Jan	$\frac{1,200}{12} \times .957 = 96$	July	$\frac{1,200}{12} \times 1.117 = 112$
Feb	$\frac{1,200}{12} \times .851 = 85$	Aug	$\frac{1,200}{12} \times 1.064 = 106$
Mar	$\frac{1,200}{12} \times .904 = 90$	Sept	$\frac{1,200}{12} \times .957 = 96$
Apr	$\frac{1,200}{12} \times 1.064 = 106$	Oct	$\frac{1,200}{12} \times .851 = 85$
May	$\frac{1,200}{12} \times 1.309 = 131$	Nov	$\frac{1,200}{12} \times .851 = 85$
June	$\frac{1,200}{12} \times 1.223 = 122$	Dec	$\frac{1,200}{12} \times .851 = 85$

USING REGRESSION ANALYSIS FOR FORECASTING

- We can use the same mathematical model that we employed in the least-squares method of trend projection to perform a linear-regression analysis.
- The dependent variables that we want to forecast will still be y . But now the independent variable, x , need no longer be time.
- We use the equation: $y = a + bx$
where y = value of the dependent variable (in our example, sales) a = y -axis intercept b = slope of the regression line x = independent variable

We now deal with the same mathematical model that we saw earlier, the least-squares method. But we use any potential “cause-and-effect”

Trend projections

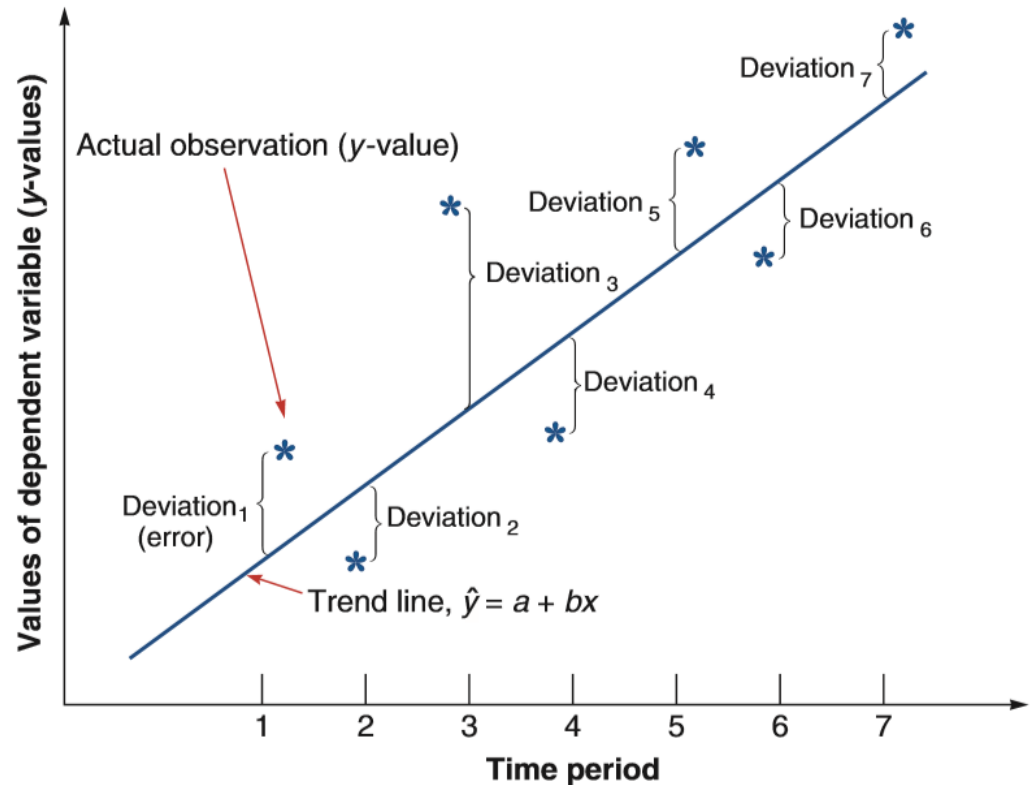
And a is = (average y) – b (average x)

$$\hat{y} = a + bx$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

Figure 4.4

The Least-Squares Method for Finding the Best-Fitting Straight Line, Where the Asterisks Are the Locations of the Seven Actual Observations or Data Points



Least Squares Example

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

Least Squares Example

YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3,063$

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \quad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

Least Squares Example

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

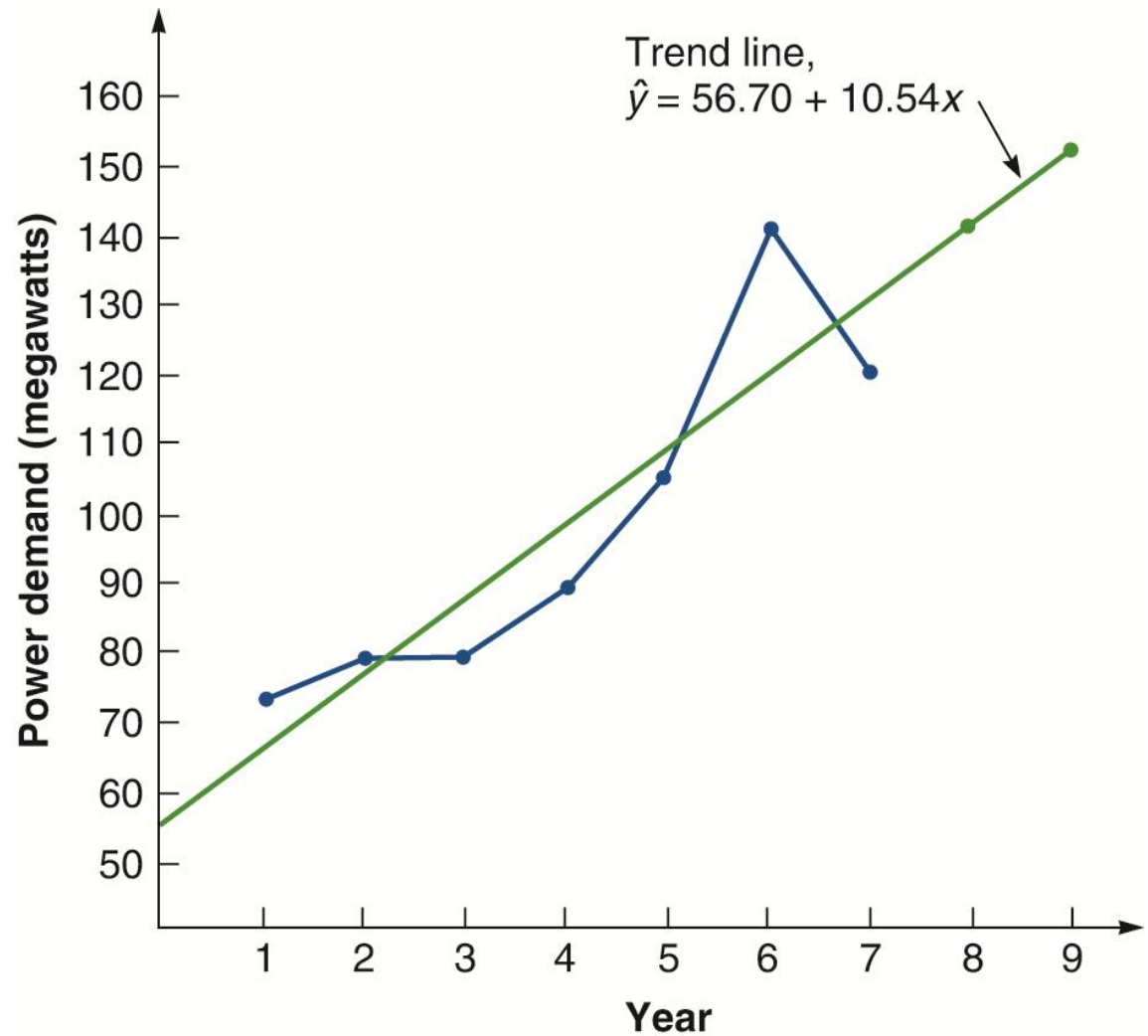
$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

$$\text{Thus, } \hat{y} = 56.70 + 10.54x$$

$$\begin{aligned}\text{Demand in year 8} &= 56.70 + 10.54(8) \\ &= 141.02, \text{ or } 141 \text{ megawatts}\end{aligned}$$

Figure 4.5

Least Squares Example



Least Squares Requirements



- We always plot the data to insure a linear relationship
- We do not predict time periods far beyond the database
- Deviations around the least squares line are assumed to be random

Forecast errors

$$\begin{aligned}\text{Tracking signal} &= \frac{\text{Cumulative error}}{\text{MAD}} \\ &= \frac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\text{MAD}}\end{aligned}$$

where

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n}$$

Monitoring and controlling forecast

- ▶ Using a tracking signal is a good way to make sure the forecasting system is continuing to do a good job
- ▶ even negative or positive

Choosing a Method

Forecast Error

MEASURE	MEANING	EQUATION
Mean absolute deviation (MAD)	How much the forecast missed the target	$\text{MAD} = \frac{\sum \text{Actual} - \text{Forecast} }{n}$
Mean squared error (MSE)	The square of how much the forecast missed the target	$\text{MSE} = \frac{\sum (\text{Forecast errors})^2}{n}$
Mean absolute percent error (MAPE)	The average percent error	$\text{MAPE} = \frac{\sum_{i=1}^n 100 \text{Actual}_i - \text{Forecast}_i / \text{Actual}_i}{n}$

$$\begin{aligned}\text{Forecast error} &= \text{Actual demand} - \text{Forecast value} \\ &= A_t - F_t\end{aligned}$$

Several measures are used in practice to calculate the overall forecast error. These measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE). We now describe and give an example of each.

Mean Absolute Deviation The first measure of the overall forecast error for a model is the [mean absolute deviation \(MAD\)](#). This value is computed by taking the sum of the absolute values of the individual forecast errors (deviations) and dividing by the number of periods of data (n):

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n} \quad (4-5)$$

FORECAST ERRORS MAD

MAPE

DETERMINING THE MEAN ABSOLUTE PERCENT ERROR (MAPE)

The Port of Baltimore wants to now calculate the MAPE when $\alpha = .10$.

APPROACH ► Equation (4-7) is applied to the forecast data computed in Example 4.

SOLUTION ►

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR $100(\text{ERROR} /\text{ACTUAL})$
1	180	175.00	$100(5/180) = 2.78\%$
2	168	175.50	$100(7.5/168) = 4.46\%$
3	159	174.75	$100(15.75/159) = 9.90\%$
4	175	173.18	$100(1.82/175) = 1.05\%$
5	190	173.36	$100(16.64/190) = 8.76\%$
6	205	175.02	$100(29.98/205) = 14.62\%$
7	180	178.02	$100(1.98/180) = 1.10\%$
8	182	178.22	$100(3.78/182) = 2.08\%$
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

INSIGHT ► MAPE expresses the error as a percent of the actual values, undistorted by a single large value.

Monitoring and Controlling Forecasts

- **Tracking Signal**
- Measures **how well the forecast is predicting actual values**
- Ratio of cumulative forecast errors to mean absolute deviation (MAD)
 - Good tracking signal has low values
 - If forecasts are continually high or low, the forecast has a bias error

Monitoring and Controlling Forecasts (2 of 2)

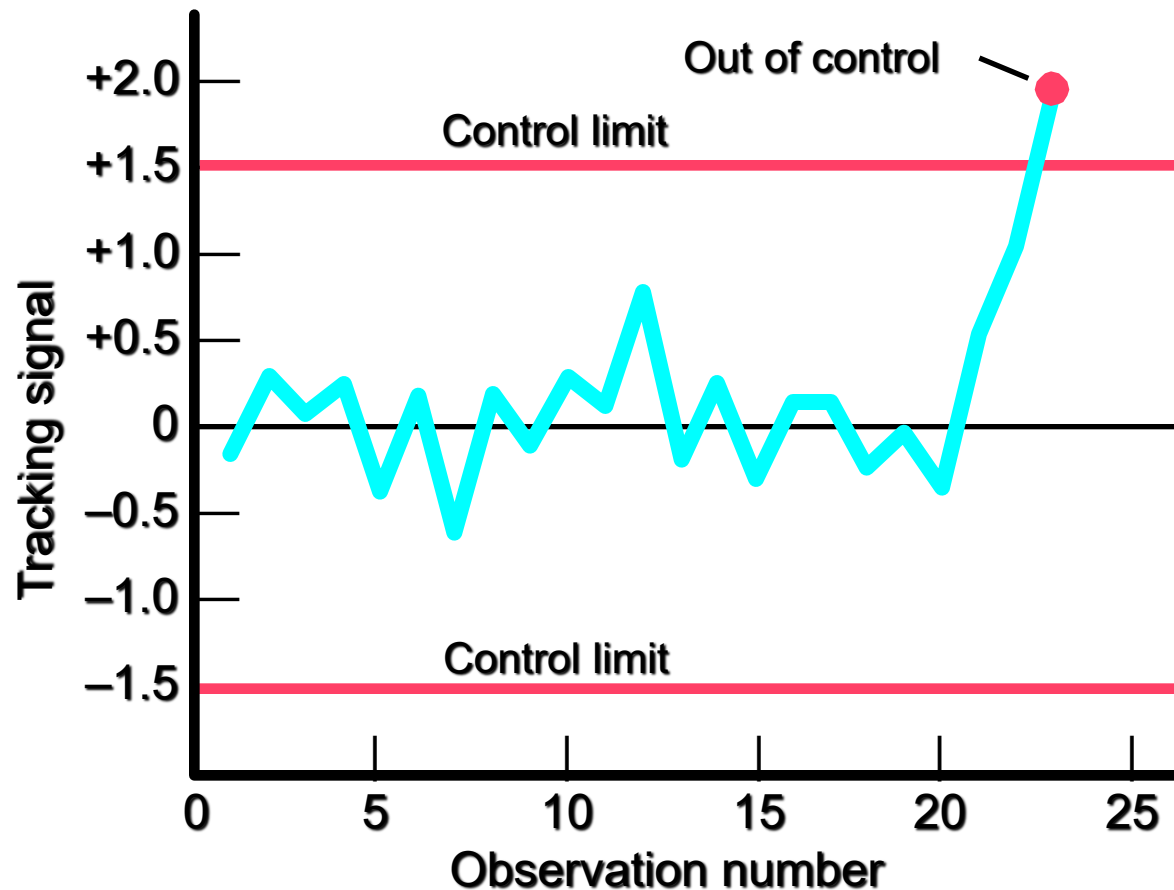
$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}}$$

$$= \frac{\sum (\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\frac{\sum |\text{Actual} - \text{Forecast}|}{n}}$$

Choosing a Method

Tracking Signals

$$\text{Tracking signal} = \frac{\text{CFE}}{\text{MAD}}$$



COMPARING

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	ABSOLUTE DEVIATION FOR $\alpha = .10$	FORECAST WITH $\alpha = .50$	ABSOLUTE DEVIATION FOR $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	<u>3.78</u>	186.30	<u>4.30</u>
Sum of absolute deviations:			82.45		98.62
MAD = $\frac{\sum \text{Deviations} }{n}$			10.31		12.33

Forecasting in the service sector

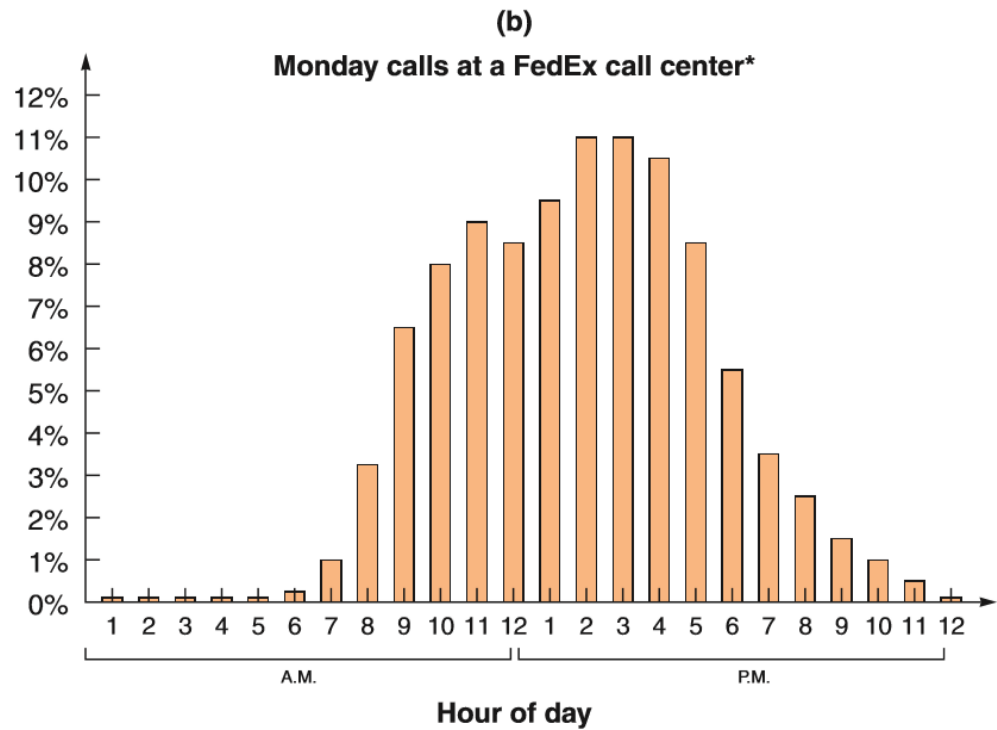
Forecasting at McDonald's, FedEx, and Walmart is as important and complex as it is for manufacturers such as Toyota and Dell.

Specialty retail facilities, such as flower shops, may have other unusual demand patterns, and those patterns will differ depending on the holiday

Fast-food restaurants are well aware not only of weekly, daily, and hourly but even 15-minute variations in demands that influence sales. Therefore, detailed forecasts of demand are needed

Services again

- Taco Bell now use point-of-sale computers that track sales every quarter hour. Taco Bell found that a 6-week moving average was the forecasting technique that minimized its mean squared error (MSE) of these quarter-hour forecasts.
- Projections of customer transactions. These in turn are used by store managers to schedule staff, who begin in 15-minute increments, not 1-hour blocks as in other industries. The forecasting model has been so successful that Taco Bell has increased customer service while documenting more than \$50 million in labor cost savings in 4 years of use.



TEST

1- Forecasting time horizons include :

- a) long range.
- b) medium range.**
- c) short range.
- d) all of the above.**

2- Average demand for iPods in the Rome, Italy, Apple store is 800 units per month. The May monthly index is 1.25. What is the seasonally adjusted sales forecast for May?

- a) 640 units
- b) 798.75 units**
- c) 800 units
- d) 1,000 units**
- e) cannot be calculated with the information given**

To continue

3- Qualitative methods of forecasting include:

- a) sales force composite.
- b) jury of executive opinion.**
- c) consumer market survey.
- d) exponential smoothing.
- e) all except d**

4- The tracking signal is the:

- a) standard error of the estimate.**
- b) cumulative error.
- c) mean absolute deviation (MAD).
- d) ratio of the cumulative error to MAD.**
- e) mean absolute percent error (MAPE).**

Self test

5-The difference between a *moving-average* model and an *exponential smoothing* model is that

6-Three popular measures of forecast accuracy are:

- a) total error, average error, and mean error.
- b) average error, median error, and maximum error.
- c) median error, minimum error, and maximum absolute error.
- d) mean absolute deviation, mean squared error, and mean absolute percent error.

Application

COMPUTING THE TRACKING SIGNAL AT CARLSON'S BAKERY

Carlson's Bakery wants to evaluate performance of its croissant forecast.

APPROACH ► Develop a tracking signal for the forecast, and see if it stays within acceptable limits, which we define as ± 4 MADs.

SOLUTION ► Using the forecast and demand data for the past 6 quarters for croissant sales, we develop a tracking signal in the following table:

QUARTER	ACTUAL DEMAND	FORECAST DEMAND	ERROR	CUMULATIVE ERROR	ABSOLUTE FORECAST ERROR	CUMULATIVE ABSOLUTE FORECAST ERROR	MAD	TRACKING SIGNAL (CUMULATIVE ERROR/MAD)
1	90	100	-10	-10	10	10	10.0	-10/10 = -1
2	95	100	-5	-15	5	15	7.5	-15/7.5 = -2
3	115	100	+15	0	15	30	10.0	0/10 = 0
4	100	110	-10	-10	10	40	10.0	-10/10 = -1
5	125	110	+15	+5	15	55	11.0	+5/11 = +0.5
6	140	110	+30	+35	30	85	14.2	+35/14.2 = +2.5

$$\text{At the end of quarter 6, MAD} = \frac{\sum |\text{Forecast errors}|}{n} = \frac{85}{6} = 14.2$$

$$\text{and Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

Exercise

The following data come from regression line projections:

PERIOD	FORECAST VALUES	ACTUAL VALUES
1	410	406
2	419	423
3	428	423
4	435	440

Compute the MAD and MSE.

Another

Room registrations in the Toronto Towers Plaza Hotel have been recorded for the past 9 years. To project future occupancy, management would like to determine the mathematical trend of guest registration. This estimate will help the hotel determine whether future expansion will be needed. Given the following time-series data, develop a regression equation relating registrations to time (e.g., a trend equation). Then forecast year 11 registrations. Room registrations are in the thousands:

Year 1: 17	Year 2: 16	Year 3: 16	Year 4: 21	Year 5: 20
Year 6: 20	Year 7: 23	Year 8: 25	Year 9: 24	

Seasonal

Quarterly demand for Ford F150 pickups at a New York auto dealer is forecast with the equation:

$$\hat{y} = 10 + 3x$$

where x = quarters, and:

Quarter I of year 1 = 0

Quarter II of year 1 = 1

Quarter III of year 1 = 2

Quarter IV of year 1 = 3

Quarter I of year 2 = 4

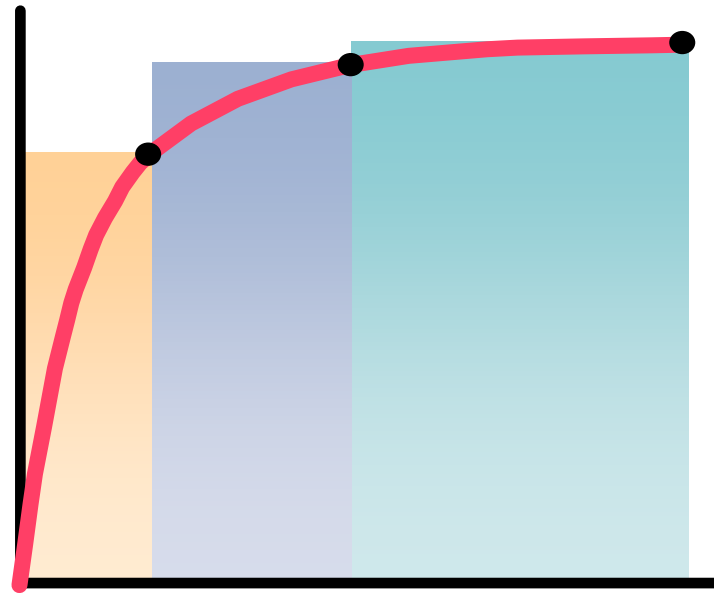
and so on

and:

\hat{y} = quarterly demand

The demand for trucks is seasonal, and the indices for Quarters I, II, III, and IV are 0.80, 1.00, 1.30, and 0.90, respectively. Forecast demand for each quarter of year 3. Then, seasonalize each forecast to adjust for quarterly variations.

Inventory Management



Inventory management why ?

- When Jeff Bezos opened his revolutionary business in 1995, **Amazon.com** was intended to be a “virtual” retailer—no inventory, **no warehouses, no overhead**—just a bunch of computers taking orders for books and authorizing others to fill them.
- Now, **Amazon stocks millions of items of inventory**, amid hundreds of thousands of bins on shelves in over 150 warehouses around the world.



Why inventory ?

1. To provide a selection of goods **for anticipated customer demand** and to separate the firm from fluctuations in that demand. Such inventories are typical in retail establishments.

2. To **decouple various parts of the production process**. For example, if a firm's supplies fluctuate, extra inventory may be necessary to decouple the production process from suppliers.

3. To take **advantage of quantity discounts**, because purchases in larger quantities may reduce the cost of goods or their delivery.

4. To hedge **against inflation and upward price changes**

Types of inventory

- **Raw material inventory**
Materials that are usually purchased but have yet to enter the manufacturing process.
- **Work-in-process (WIP) inventory**
Products or components that are no longer raw materials but have yet to become finished products.
- **Maintenance/repair/operating (MRO) inventory**
Maintenance, repair, and operating materials.

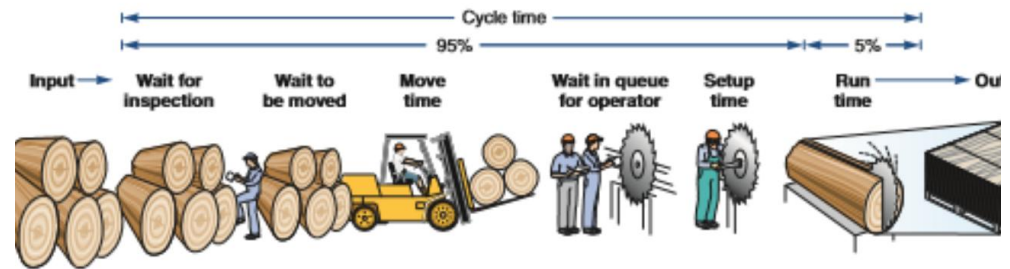


Figure 12.1

Material Flow Cycle

Most of the time that work is in-process (95% of the cycle time) is not productive time.

Inventory Costs

- Interest or Opportunity Cost
- Storage and Handling Costs
- Taxes, Insurance, and Shrinkage



Inventory Costs

- Customer Service
- Ordering Cost
- Setup Cost
- Labor and Equipment Utilization
- Transportation Costs
- Payments to Suppliers



Types of Inventory

Cycle Inventory

$$\text{Average cycle inventory} = \frac{Q + 0}{2}$$

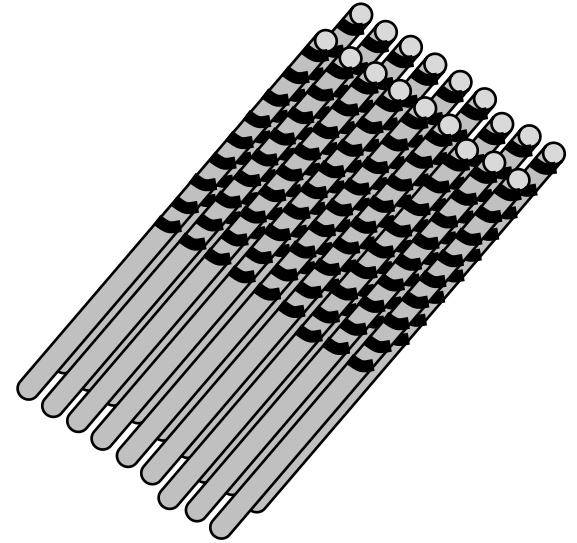
Safety Stock Inventory

Anticipation Inventory

Pipeline Inventory

$$\text{Pipeline inventory} = D_L^{\bar{}} = dL$$

Types of Inventory

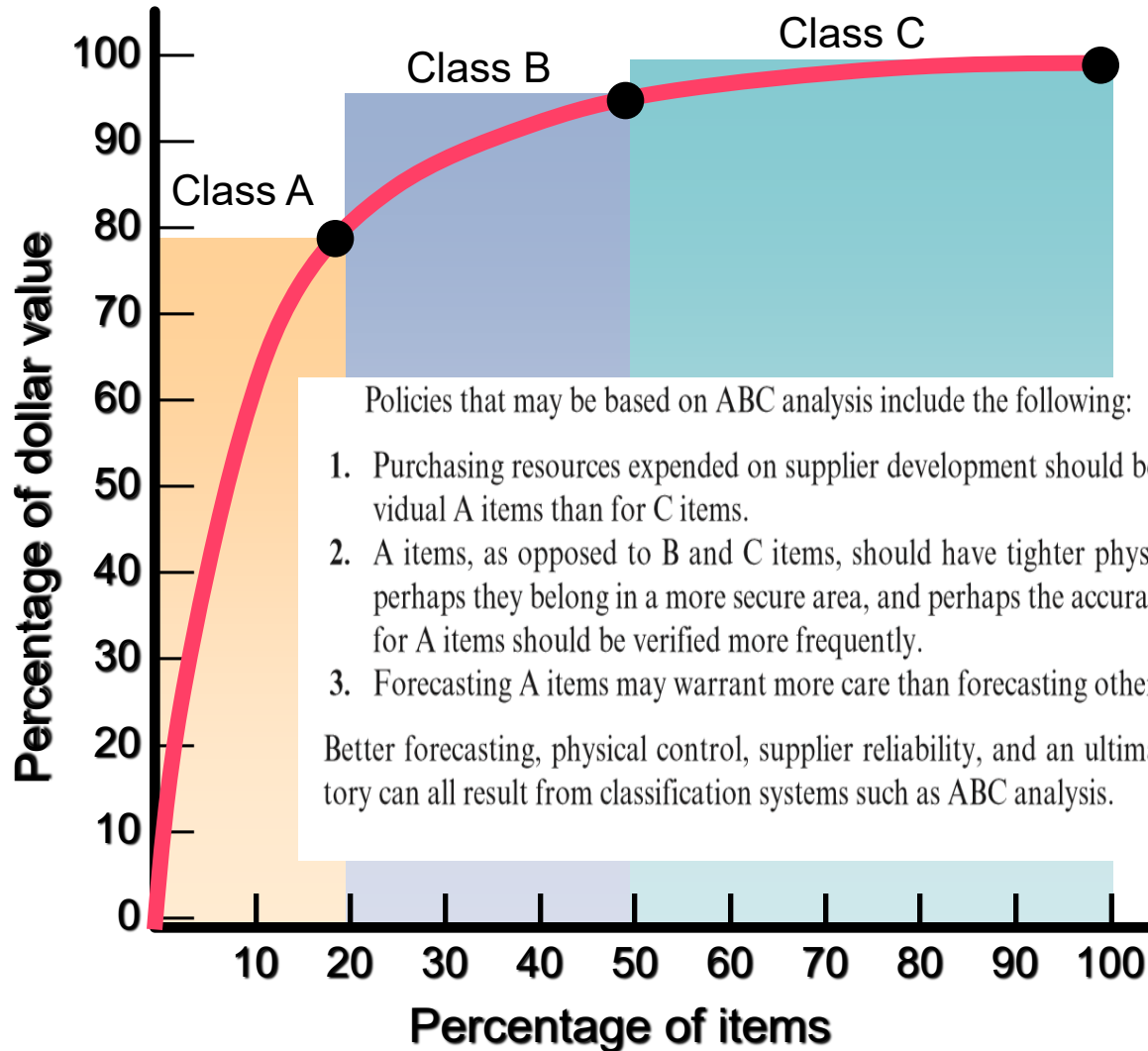


$$\begin{aligned}\text{Cycle inventory} &= Q/2 \\ &= 280/2 \\ &= 140 \text{ drills}\end{aligned}$$

$$\begin{aligned}\text{Pipeline inventory} &= \bar{D}_L = dL \\ &= (70 \text{ drills/week})(3 \text{ weeks}) \\ &= 210 \text{ drills}\end{aligned}$$

Example 15.1

ABC Analysis



How
Much?
When!



Record accuracy

And Cycle counting

- A continuing reconciliation of inventory with inventory records



In this hospital, these vertically rotating storage carousels provide rapid access to hundreds of critical items and at the same time save floor space. This Omnicell inventory management carousel is also secure and has the added advantage of printing bar code labels.

Cycle counting example (pipeline)

- CYCLE COUNTING AT COLE'S TRUCKS, INC. Cole's Trucks, Inc., a builder of high-quality refuse trucks, has about 5,000 items in its inventory. It wants to determine how many items to cycle count each day.
- APPROACH After hiring Matt Clark, a bright young OM student, for the summer, the firm determined that it has
 - 500 A items, 1,750 B items, and 2,750 C items.
- Company policy is to count all A items every month (every 20 working days), all B items every quarter (every 60 working days), and all C items every 6 months (every 120 working days). The firm then allocates some items to be counted each day.

***Economic
Order
Quantity***



Economic Order Quantity



Assumptions

- 1. Demand rate is constant**
- 2. No constraints on lot size**
- 3. Only relevant costs are holding and ordering/setup**
- 4. Decisions for items are independent from other items**
- 5. No uncertainty in lead time or supply**

Economic Order Quantity

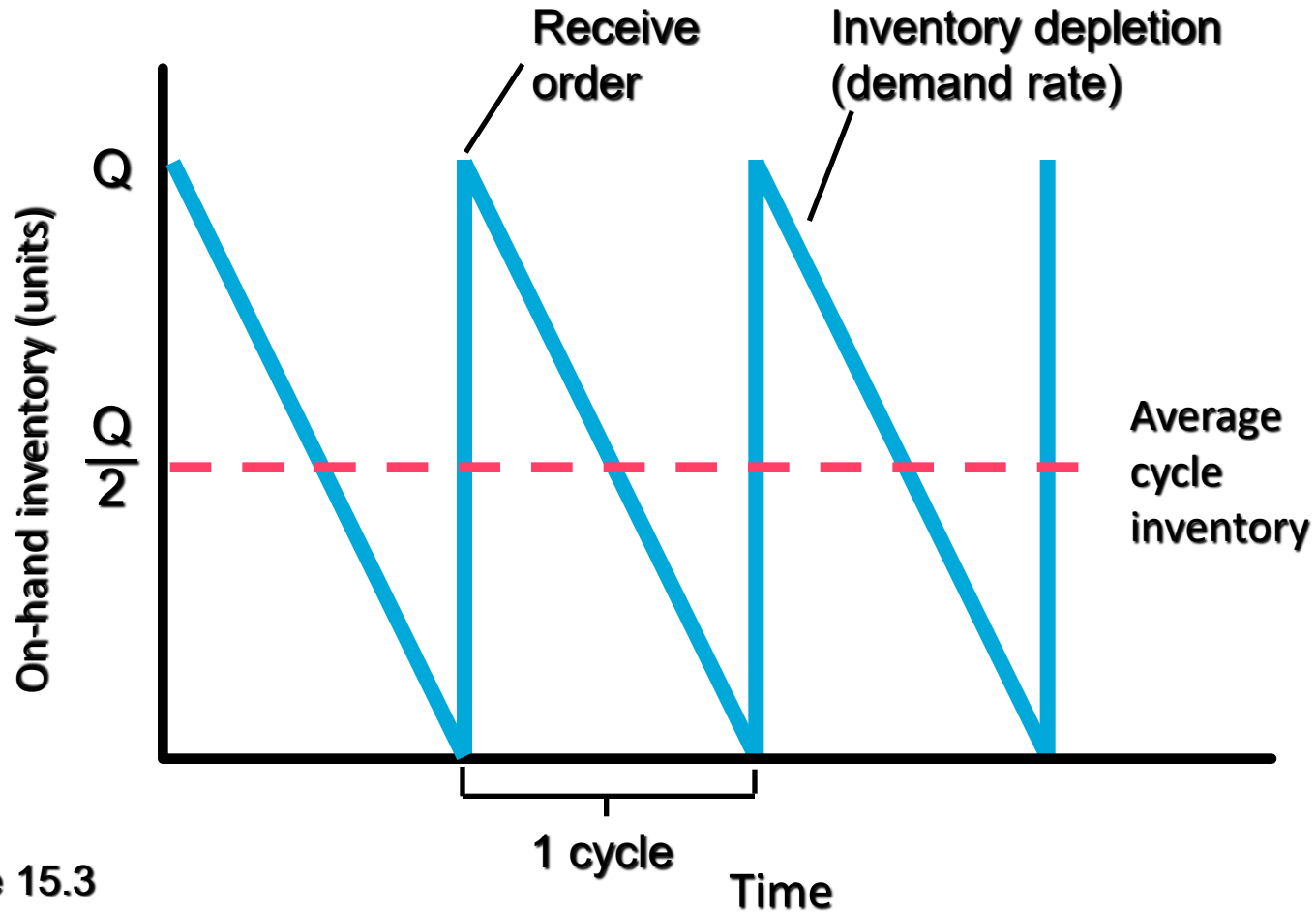


Figure 15.3

Economic Order Quantity

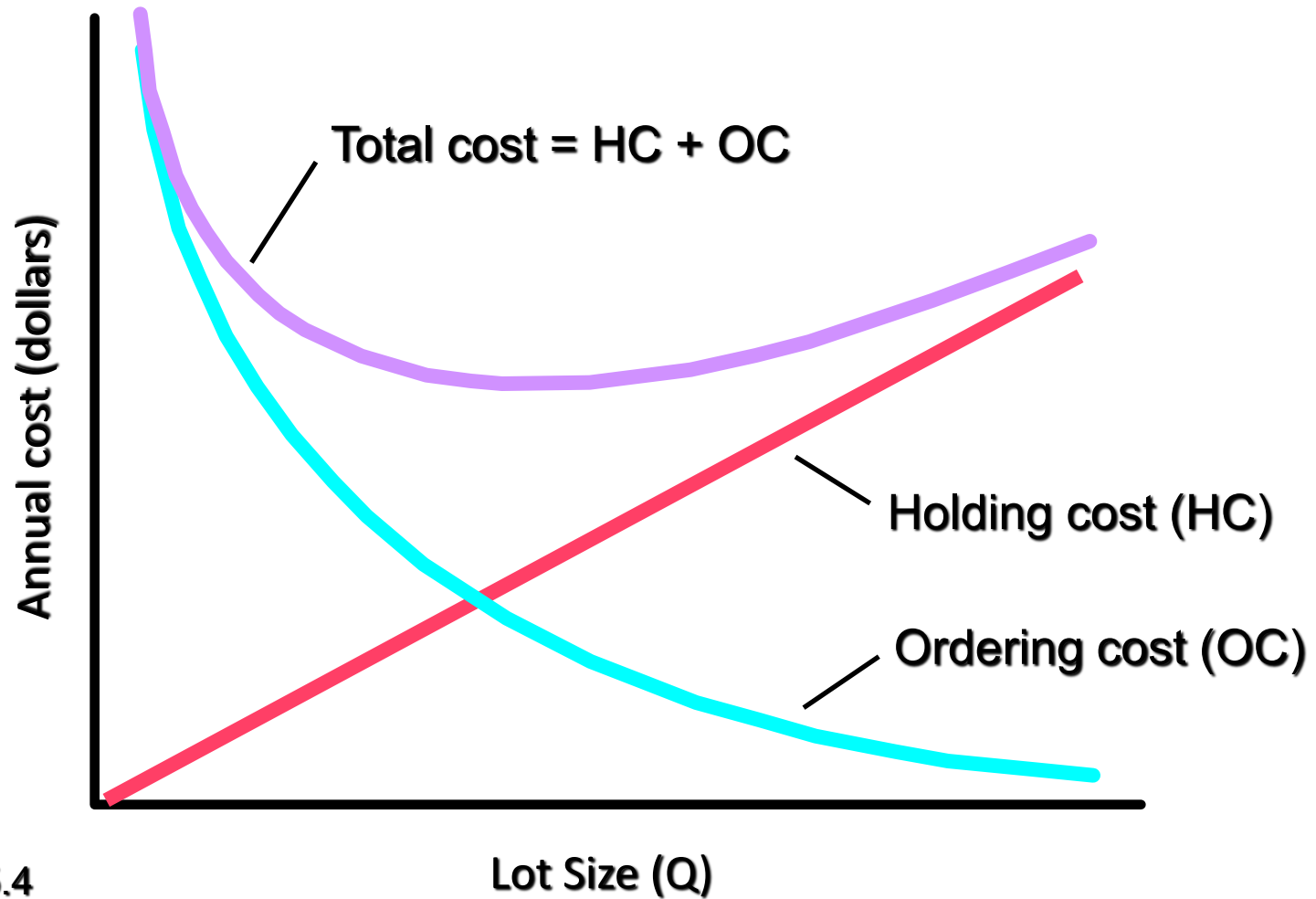


Figure 15.4

EXAMPLE

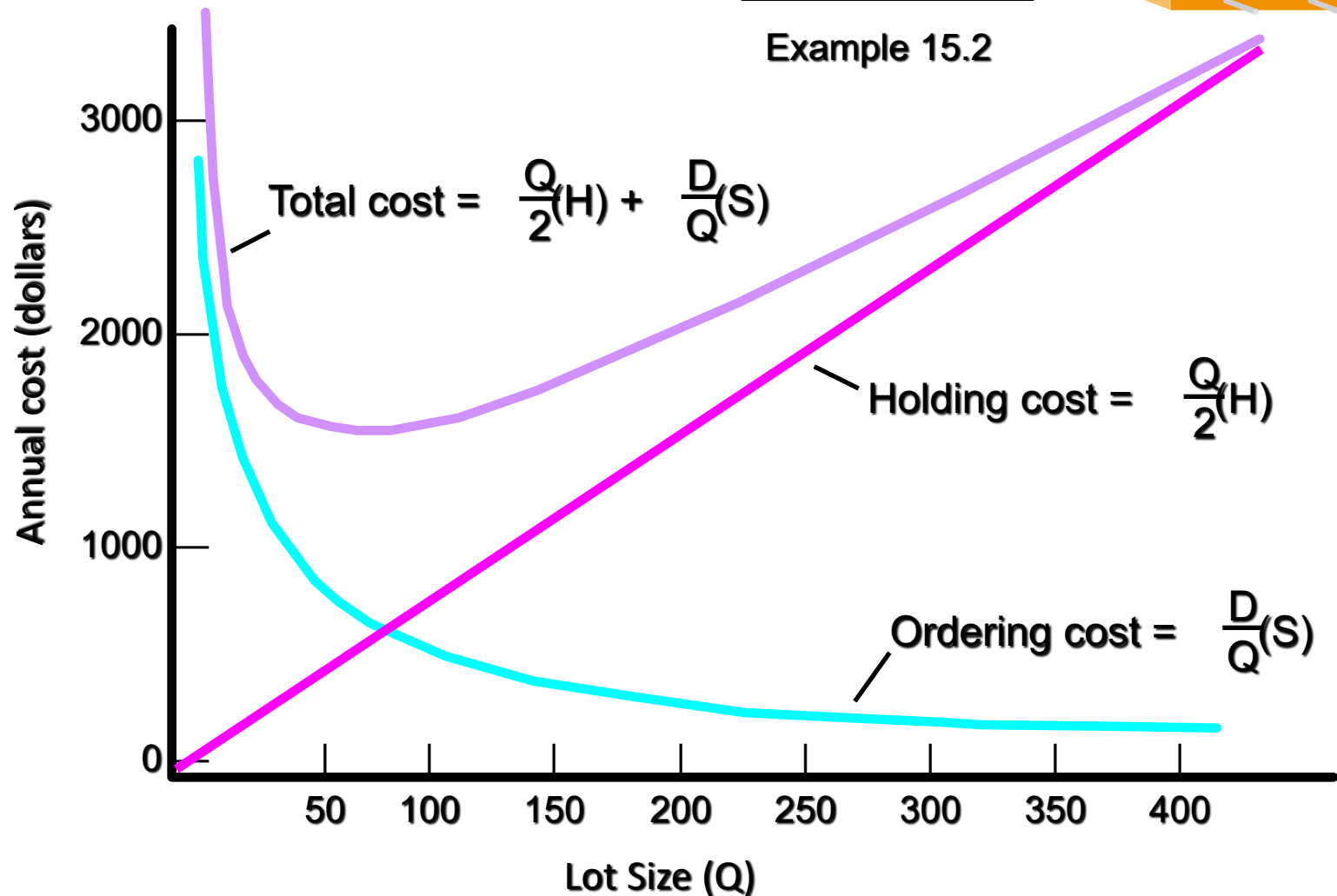
Total cost is Holding cost + ordering cost

- Costing **Out a Lot-Sizing Policy**
- A museum of natural history opened a gift shop two years ago. Managing inventories has become a problem. Low inventory turnover is squeezing profit margins and causing cash-flow *problems*.
- One of the top-selling items in the container group at the museum's gift shop is a bird feeder. Sales are 18 units per week, and the supplier charges \$60 per unit. The cost of placing an order with the supplier is \$45. Annual holding cost is 25 percent of a feeder's value, and the museum operates 52 weeks per year. Management chose a 390-unit lot size so that new orders could be placed less frequently. What is the annual cost of the current policy of using a 390-unit lot size? Would a lot size of 468 be better?
-

Economic Order Quantity



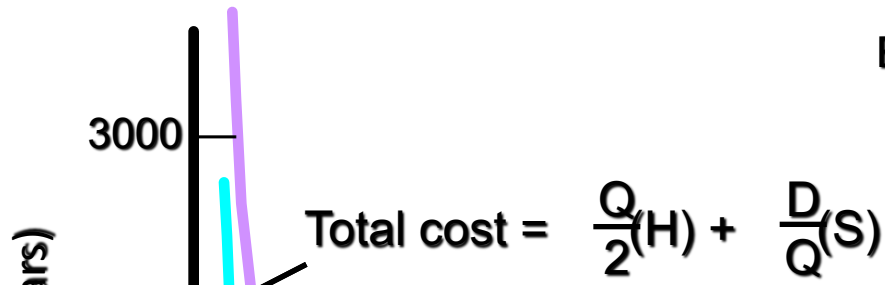
Example 15.2



Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

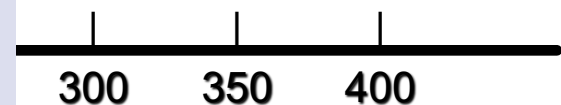
$$S = \$45 \qquad Q = 390 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

$$\text{Holding cost} = \frac{Q}{2}(H)$$

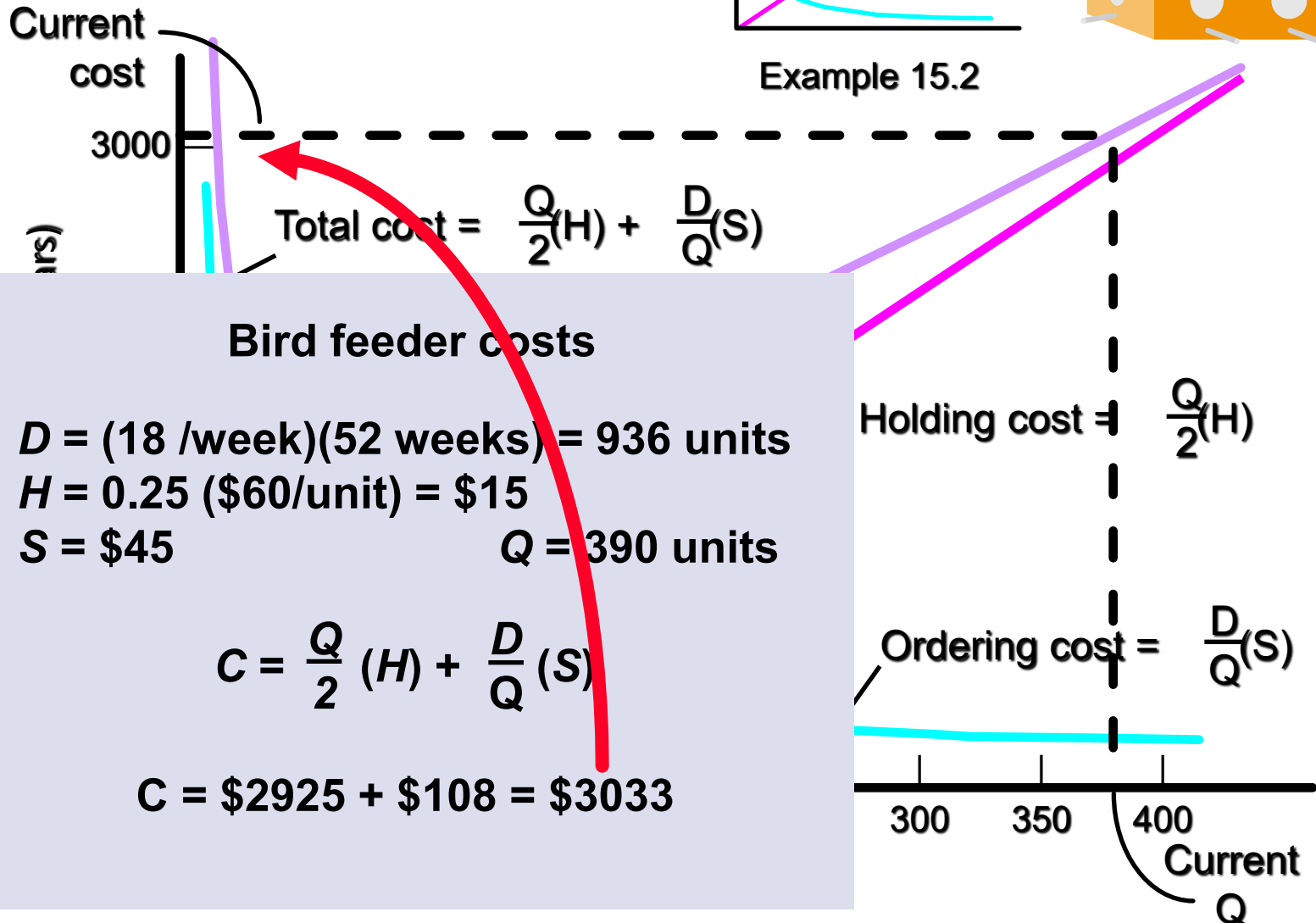
$$\text{Ordering cost} = \frac{D}{Q}(S)$$



Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

$$S = \$45 \quad Q = 390 \text{ units}$$

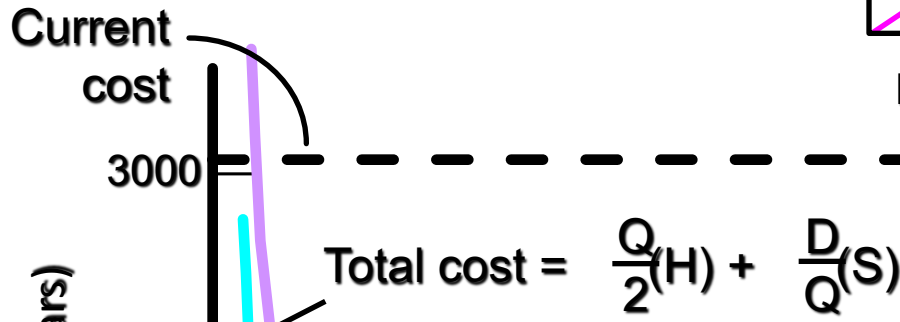
$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

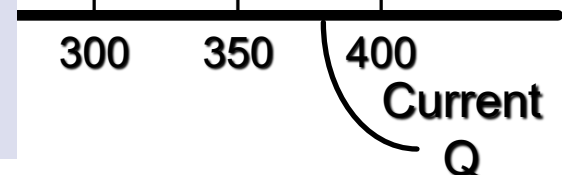
$$S = \$45 \quad Q = 390 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

$$\text{Holding cost} = \frac{Q}{2}(H)$$

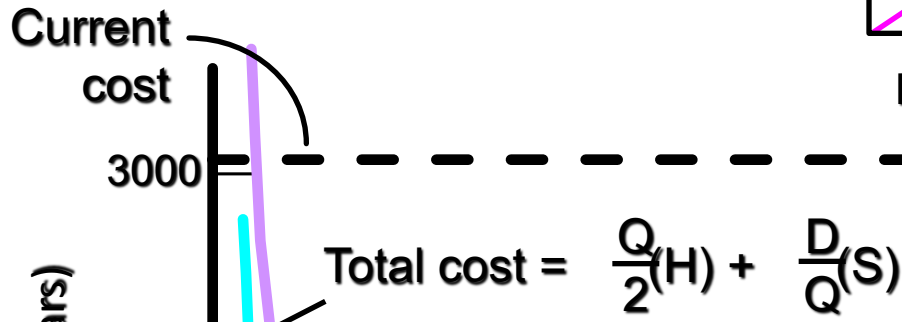
$$\text{Ordering cost} = \frac{D}{Q}(S)$$



Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

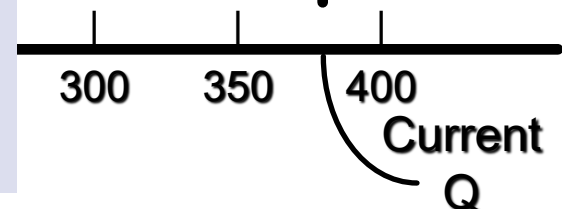
$$S = \$45 \quad Q = 468 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

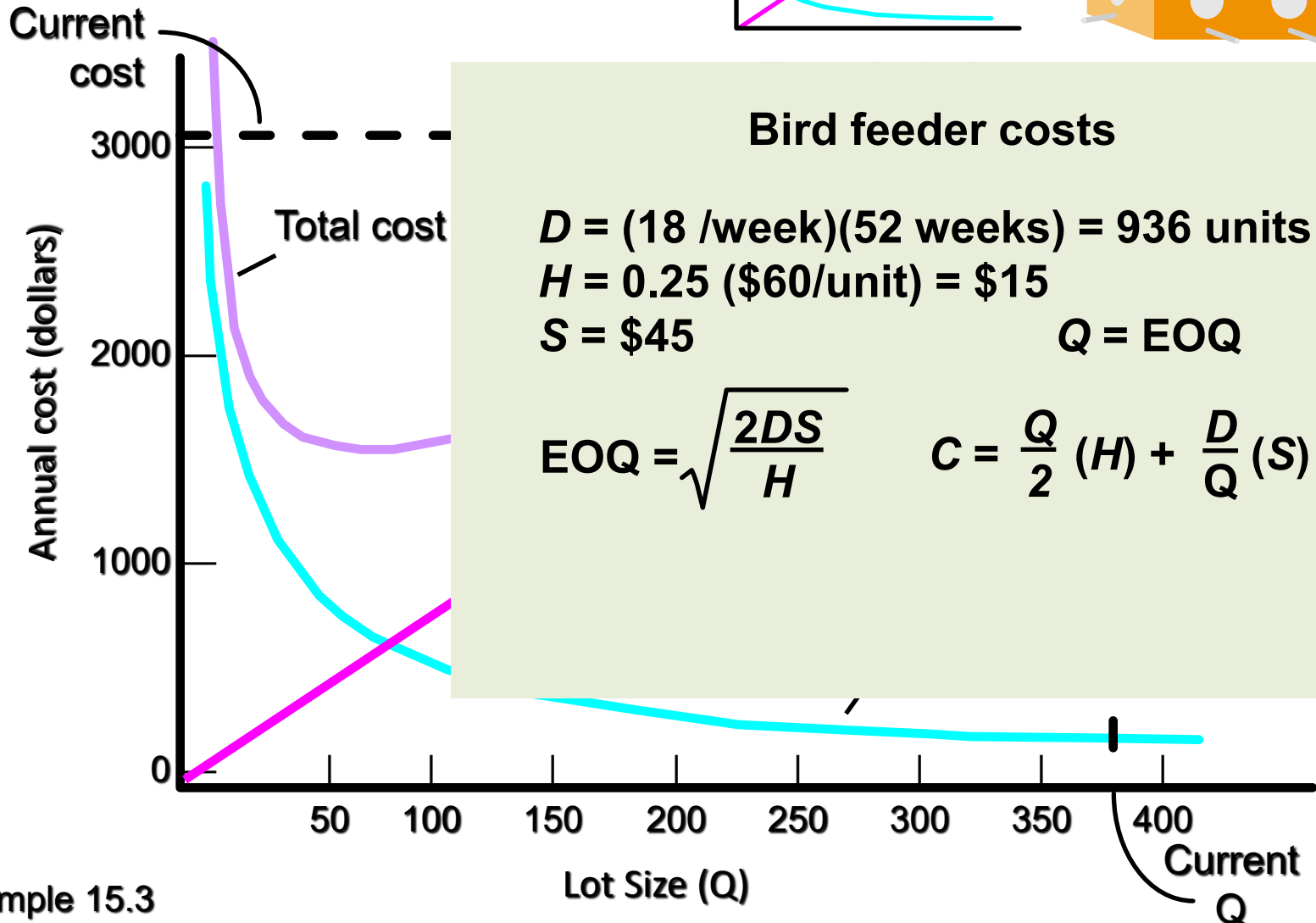
$$C = \$3510 + \$90 = \$3600$$

$$\text{Holding cost} = \frac{Q}{2}(H)$$

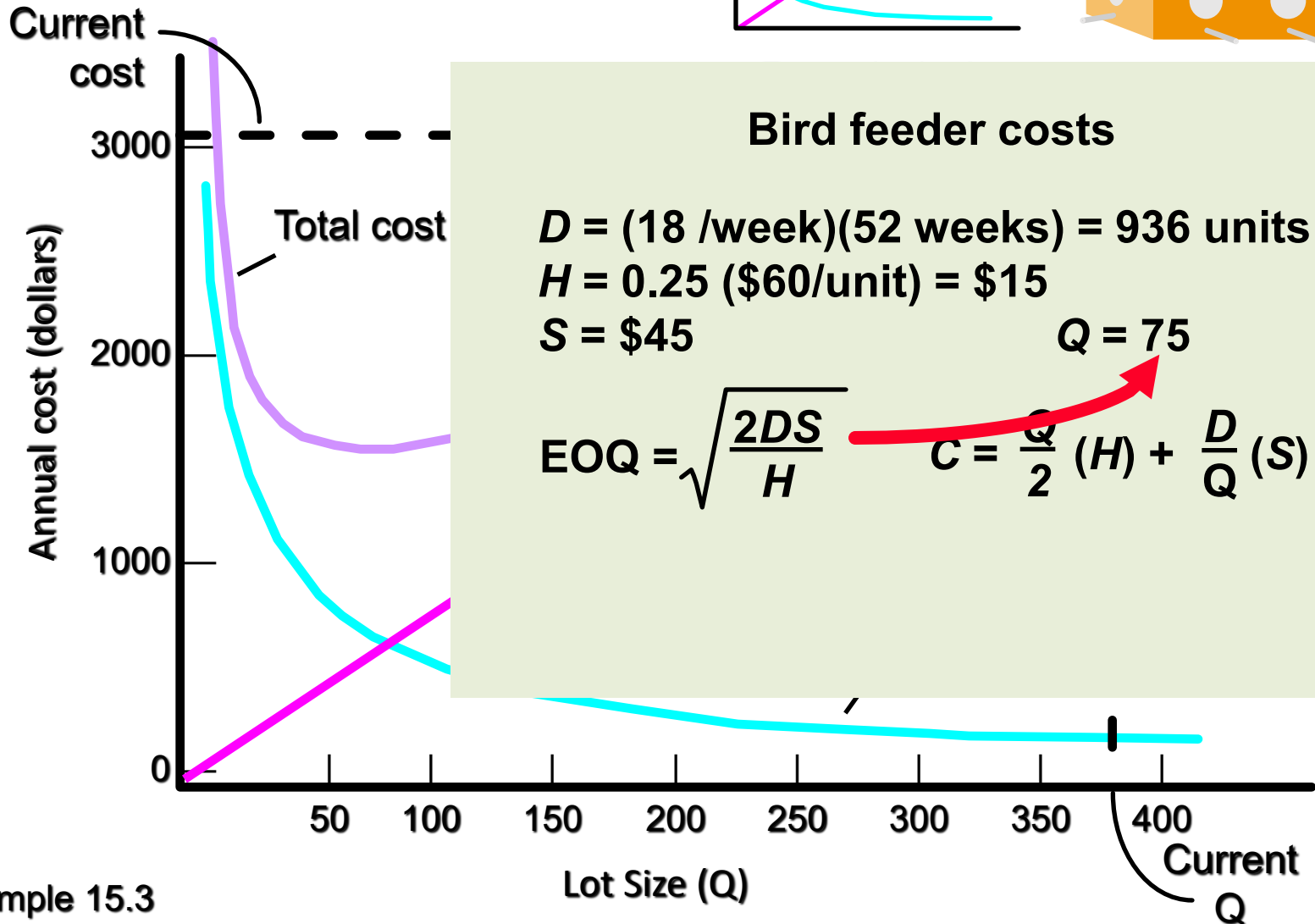
$$\text{Ordering cost} = \frac{D}{Q}(S)$$



Economic Order Quantity

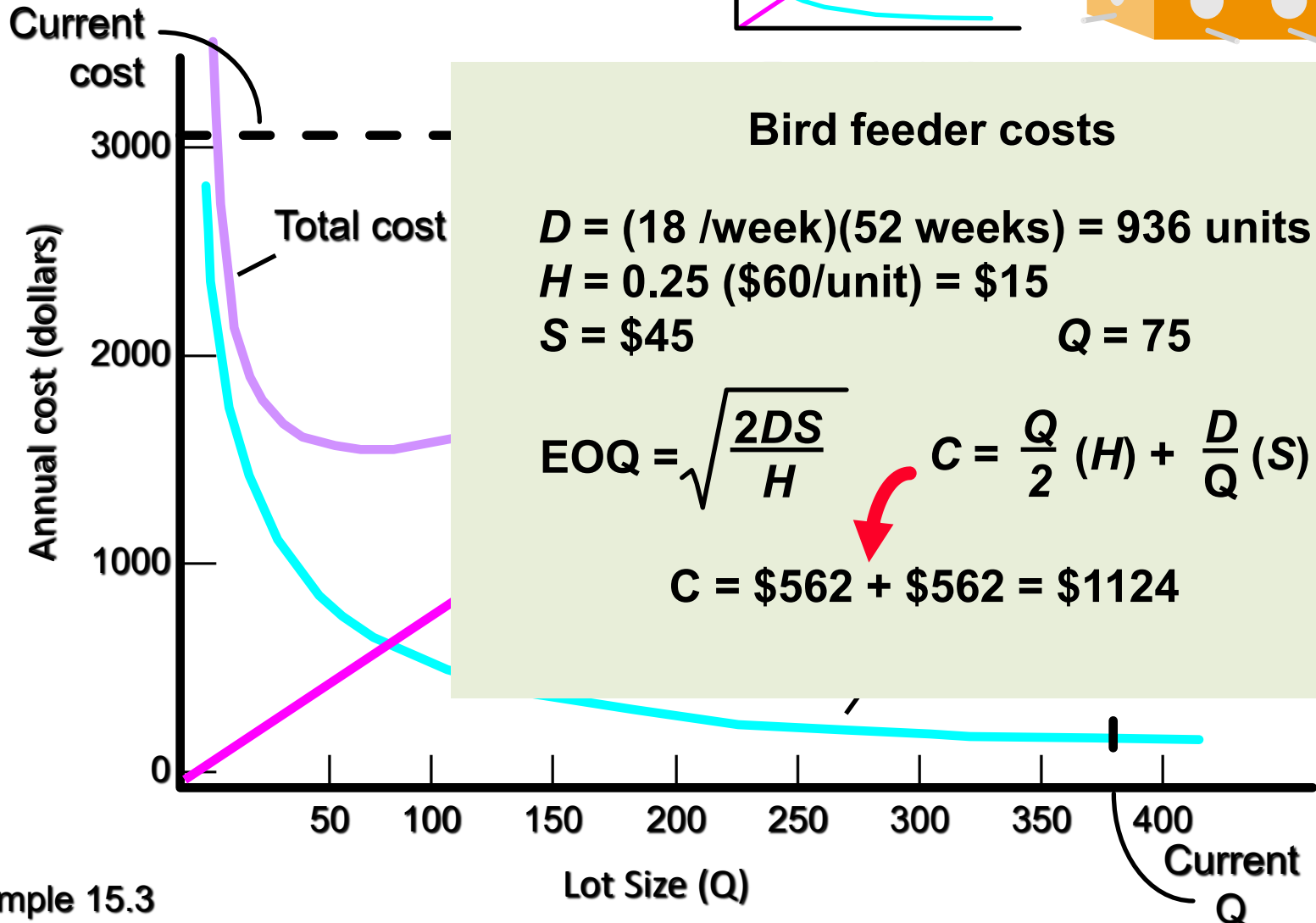


Economic Order Quantity



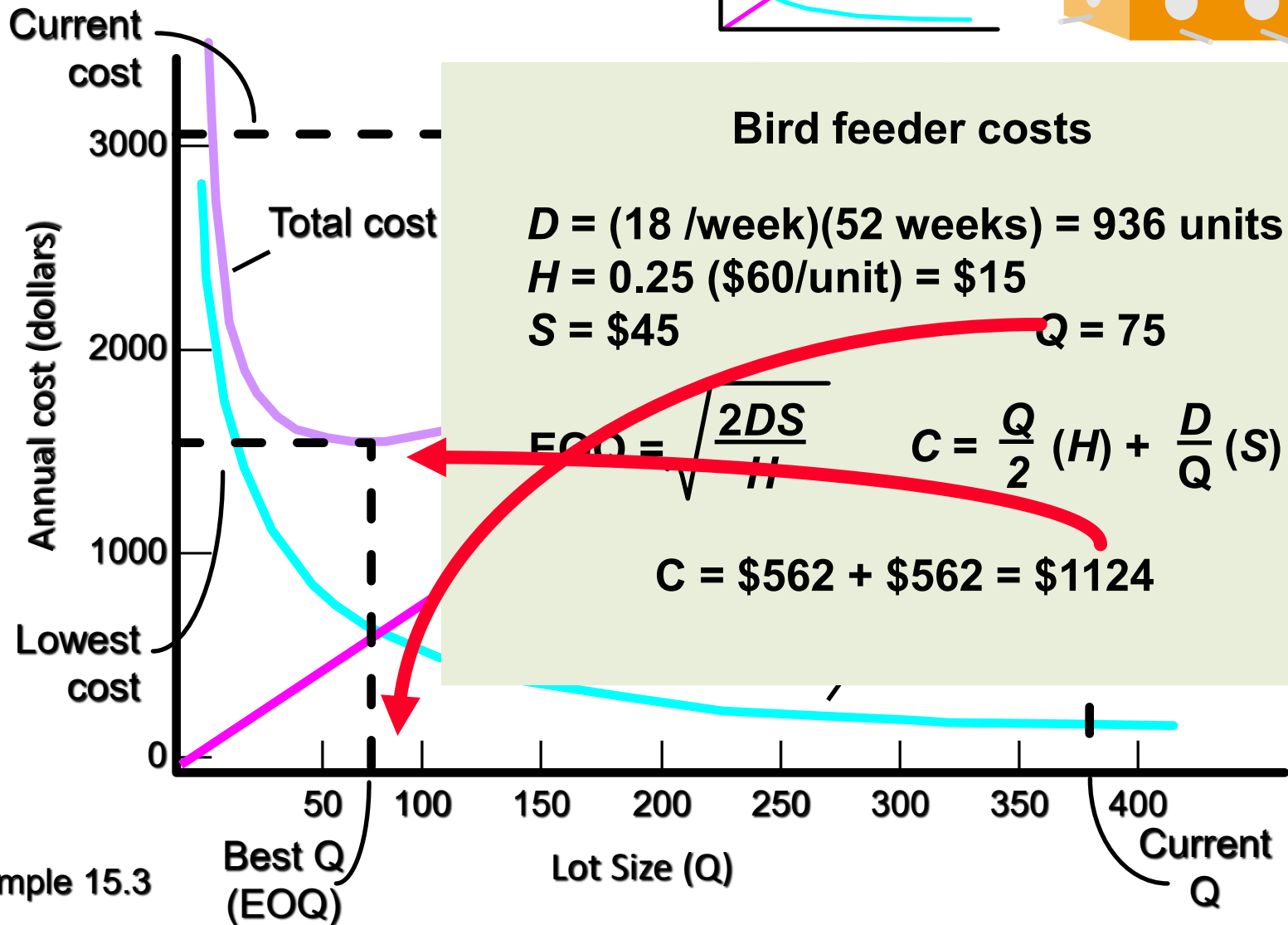
Example 15.3

Economic Order Quantity



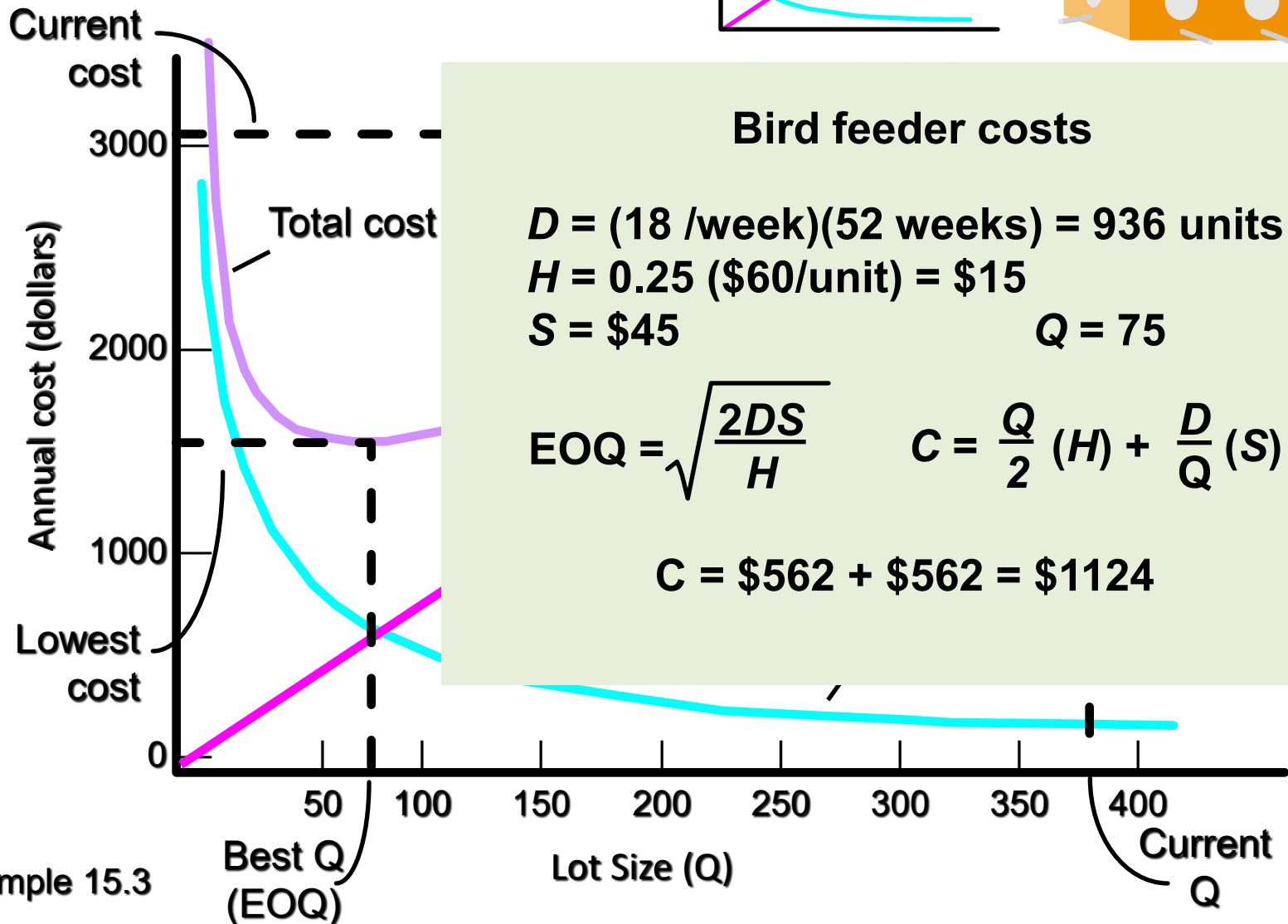
Example 15.3

Economic Order Quantity



Example 15.3

Economic Order Quantity



Economic Order Quantity



Current
cost



Bird feeder costs

Parameters

Current Lot Size (Q)	390
Demand (D)	936
Order Cost (S)	\$45
Unit Holding Cost (H)	\$15

Economic Order Quantity 75

Annual Costs

Orders per Year	2.4
Annual Ordering Cost	\$108.00
Annual Holding Cost	\$2,925.00
Annual Inventory Cost	\$3,033.00

Annual Costs based on EOQ

Orders per Year	12.48
Annual Ordering Cost	\$561.60
Annual Holding Cost	\$562.50
Annual Inventory Cost	\$1,124.10

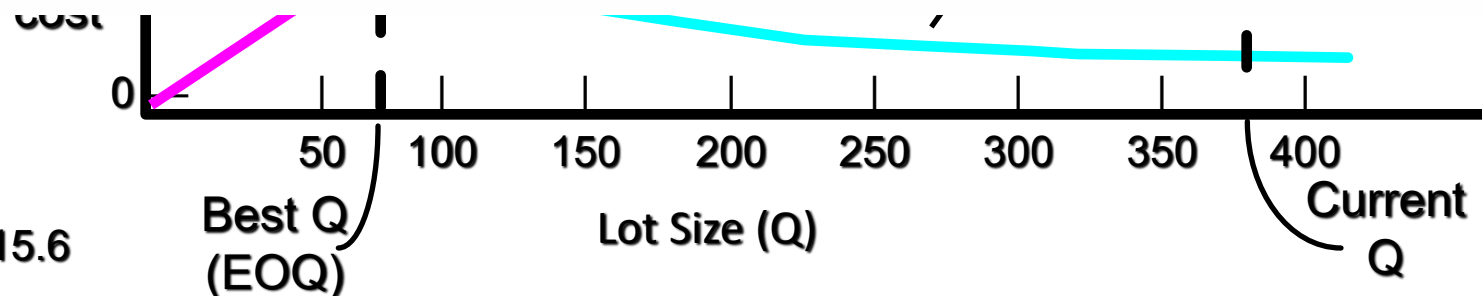
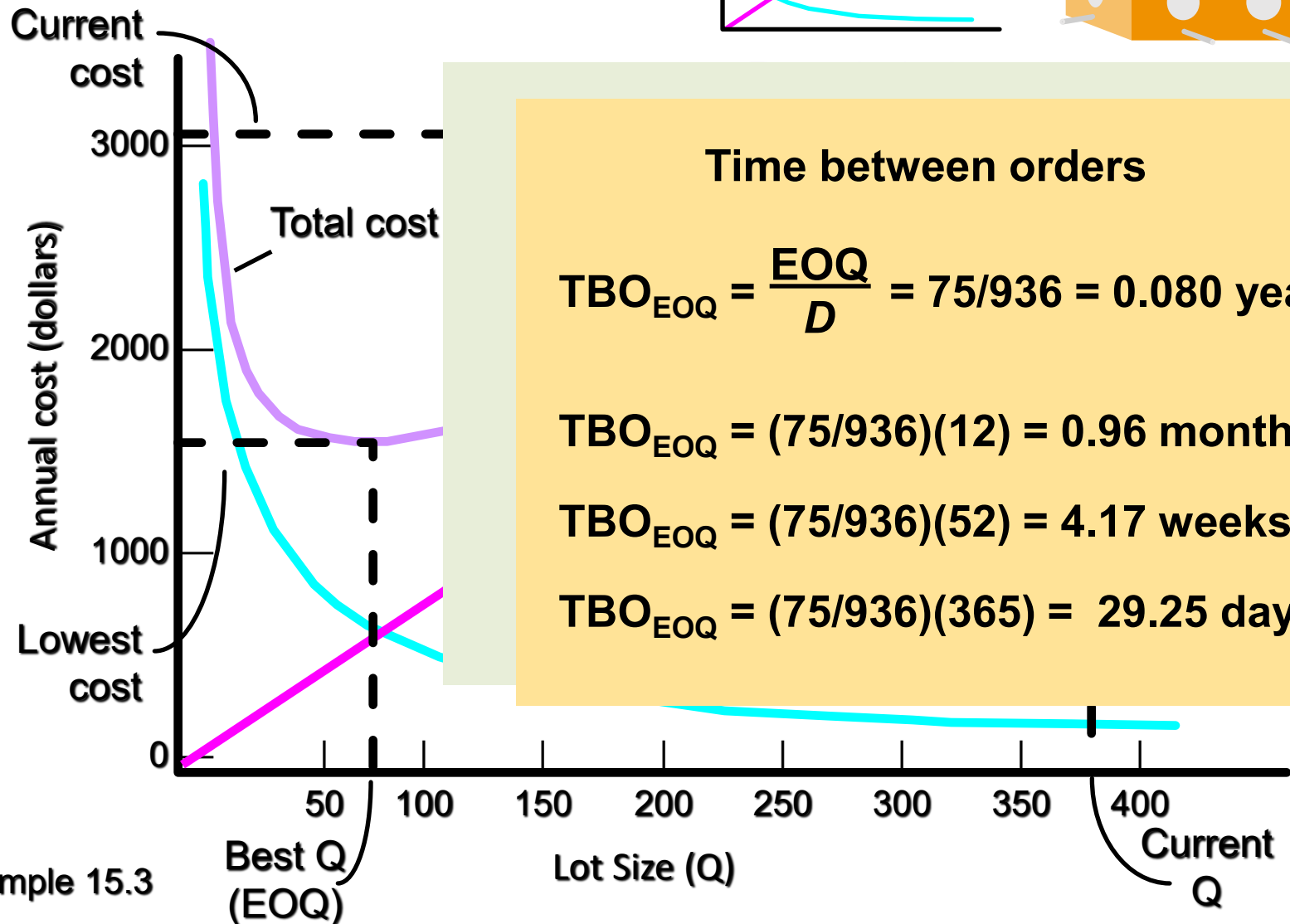


Figure 15.6

Economic Order Quantity



Example 15.3

How
Much?
When!



Continuous Review

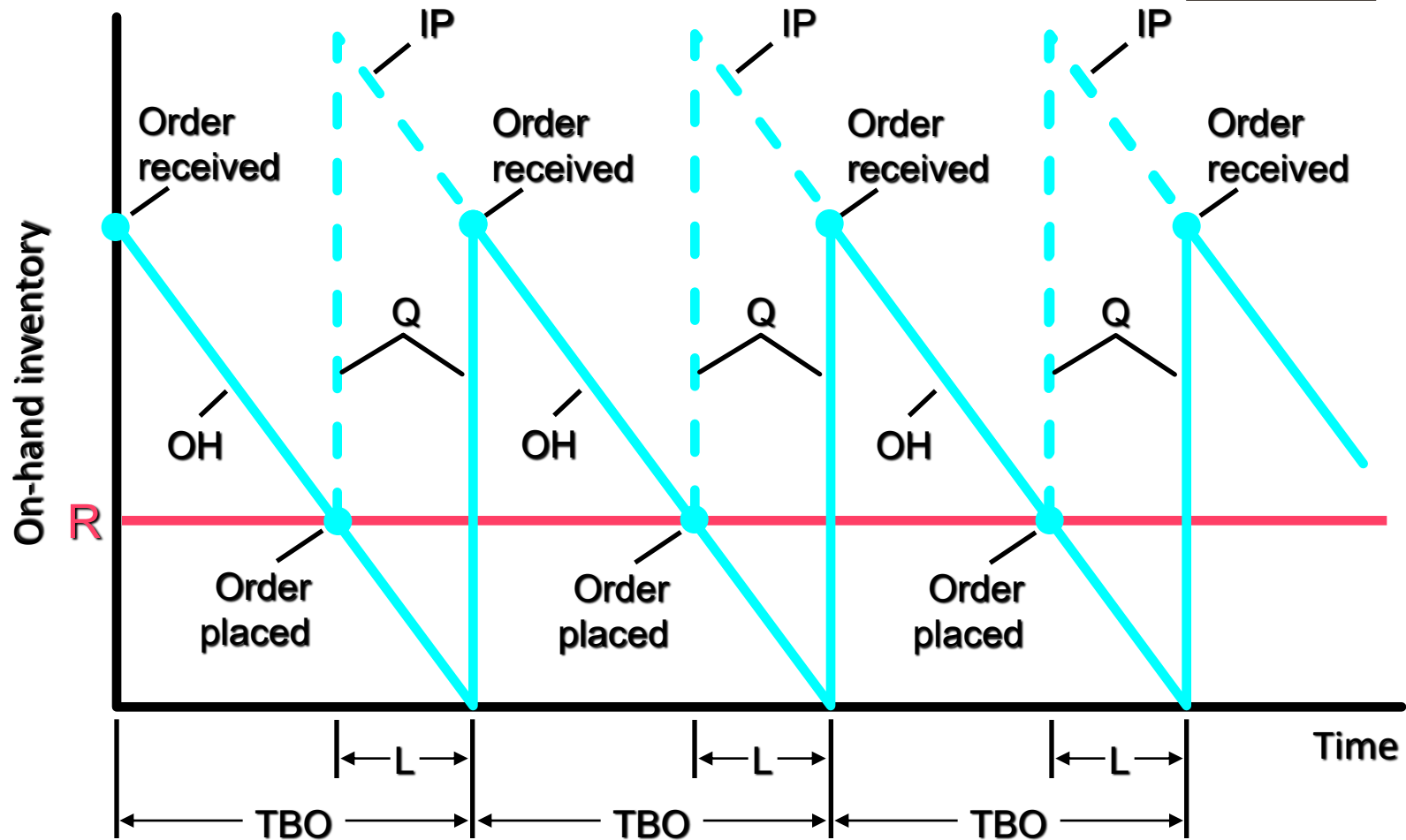


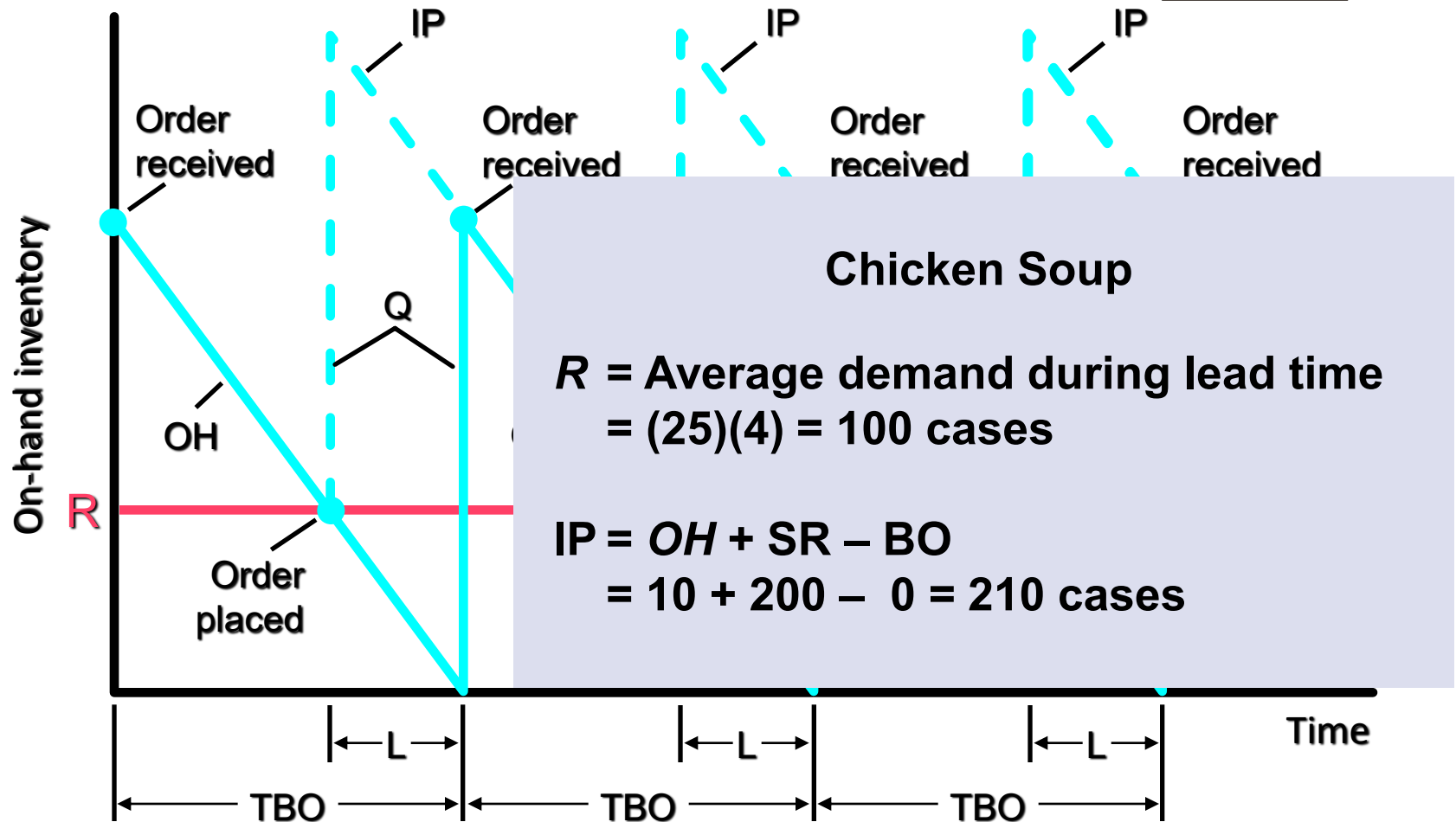
Figure 15.7

EXAMPLE

Determining Whether to Place an Order

Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always four days. The shelves were just restocked with chicken soup, leaving an on-hand inventory of only 10 cases. There are no backorders, but there is one open order for 200 cases. What is the inventory position? Should a new order be placed?

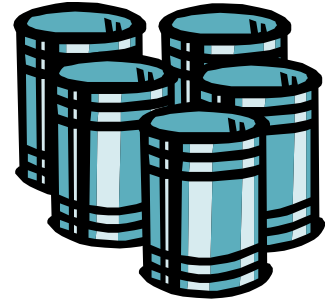
Continuous Review



Example 15.4

Special Inventory Models

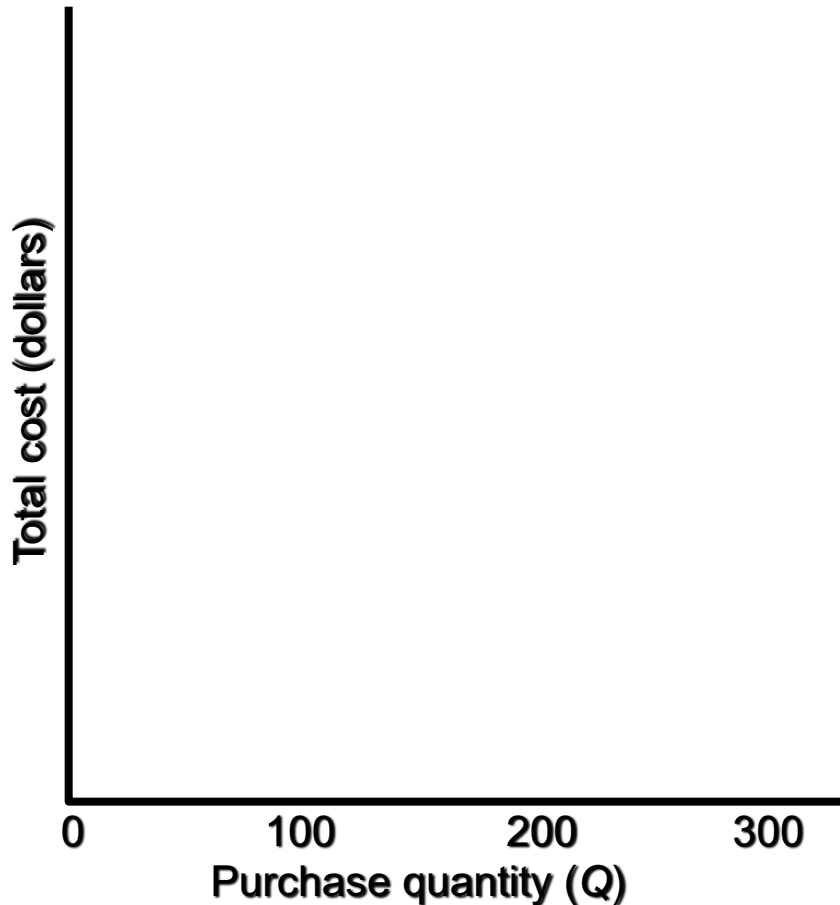
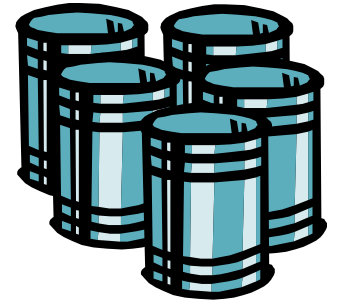
Quantity Discounts



Special Inventory Models

Quantity Discounts

Figure E.3

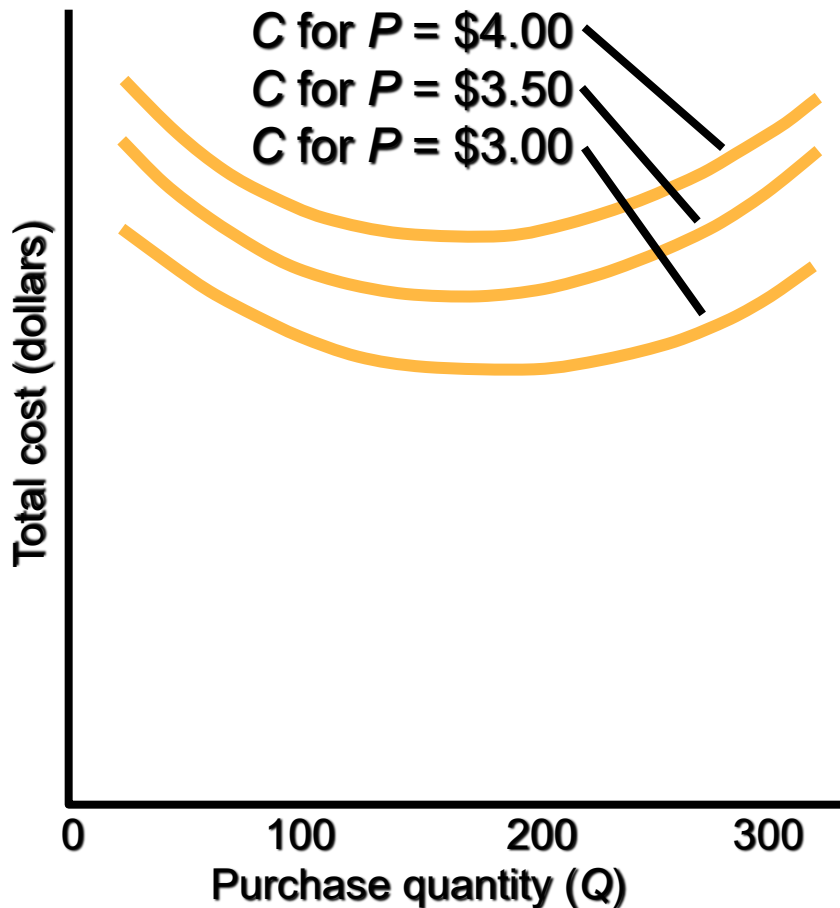
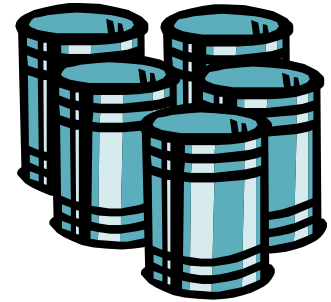


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

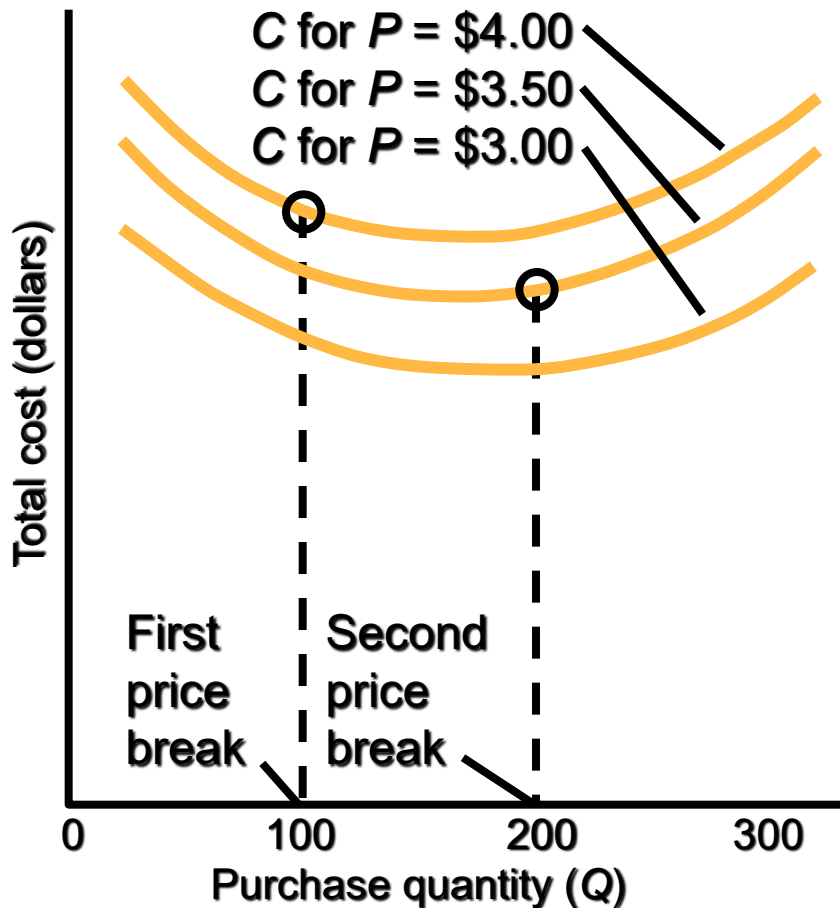
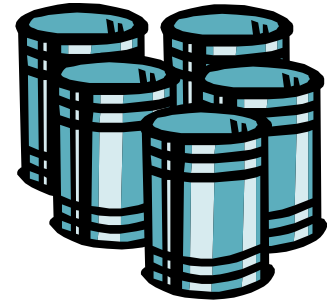


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

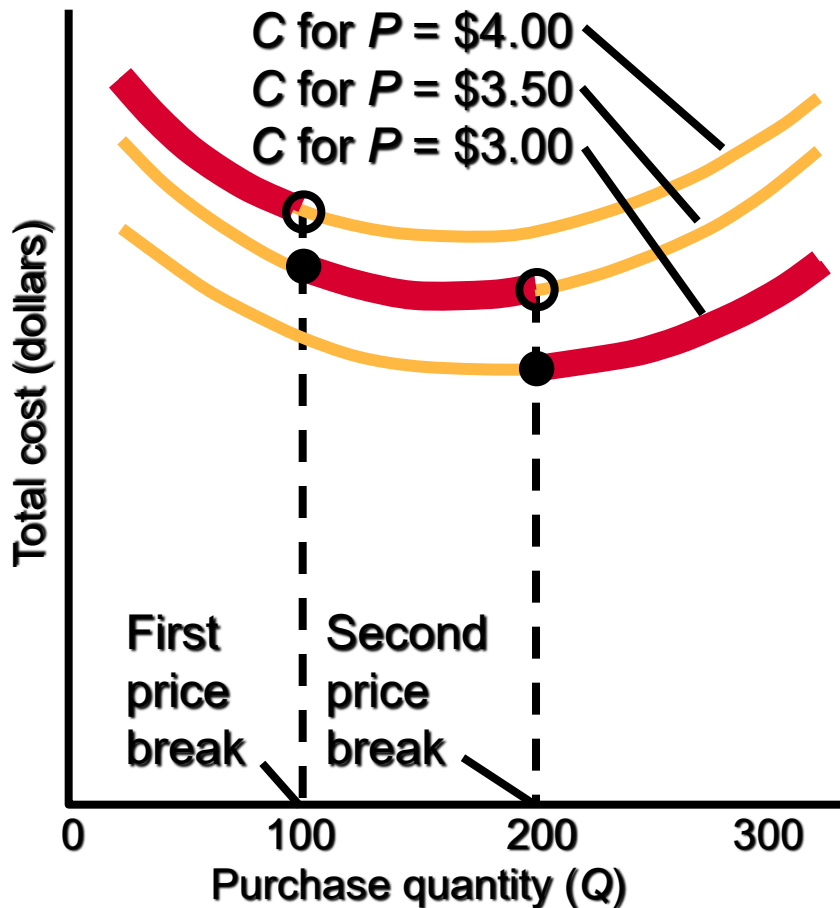
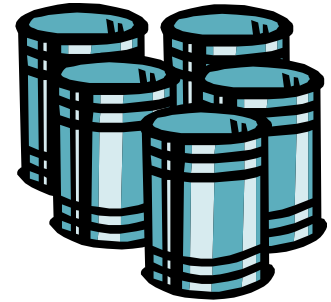


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

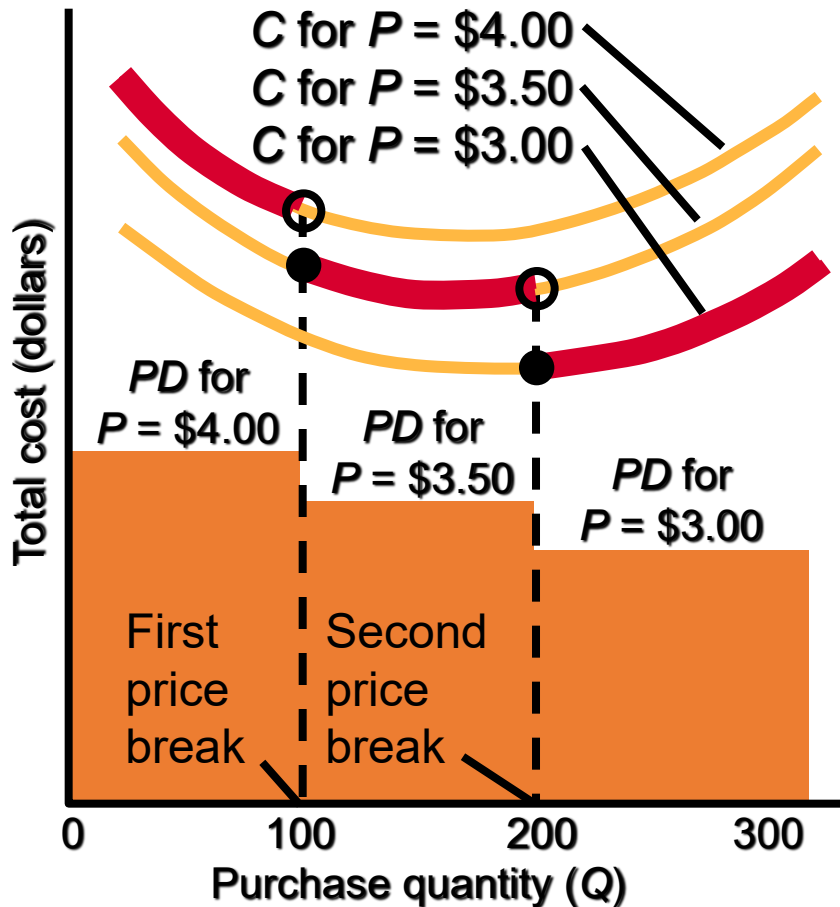
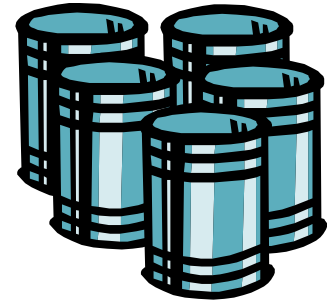


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

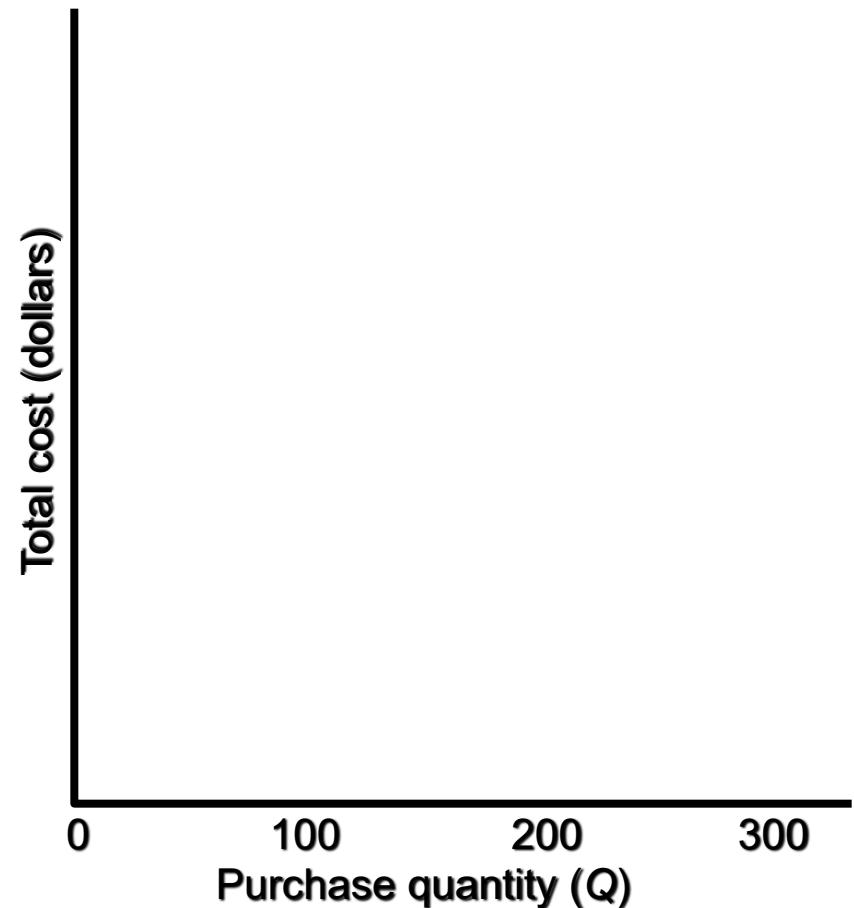
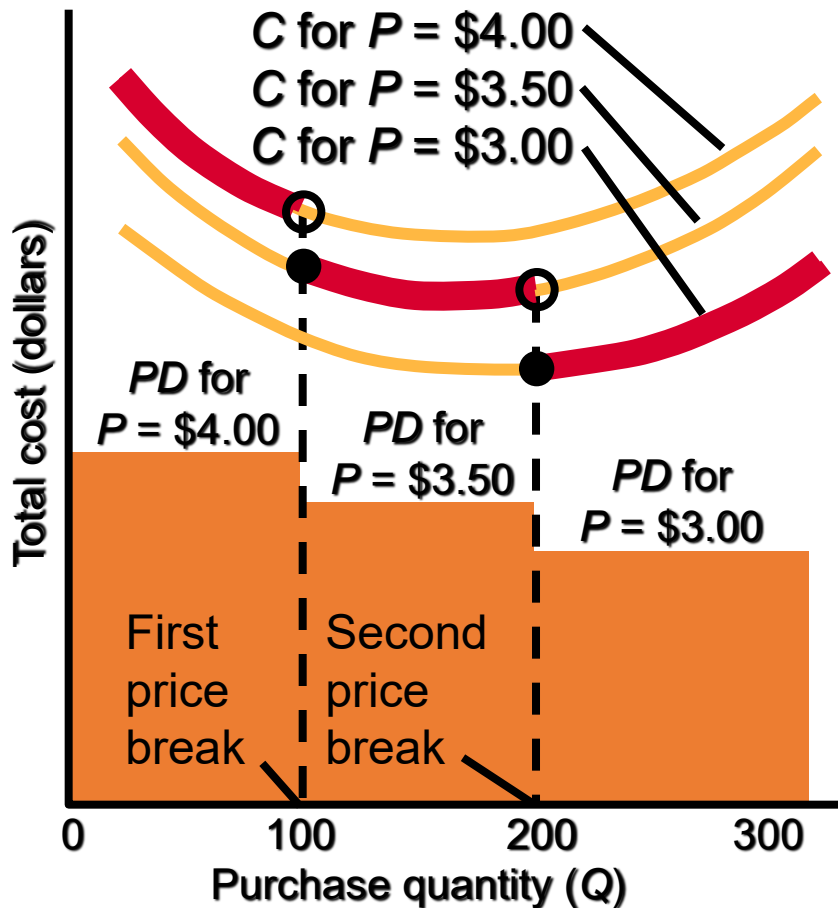
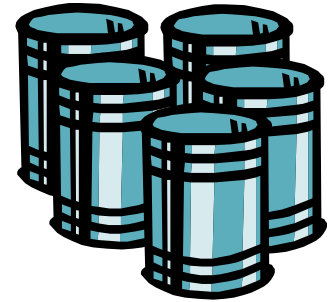


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

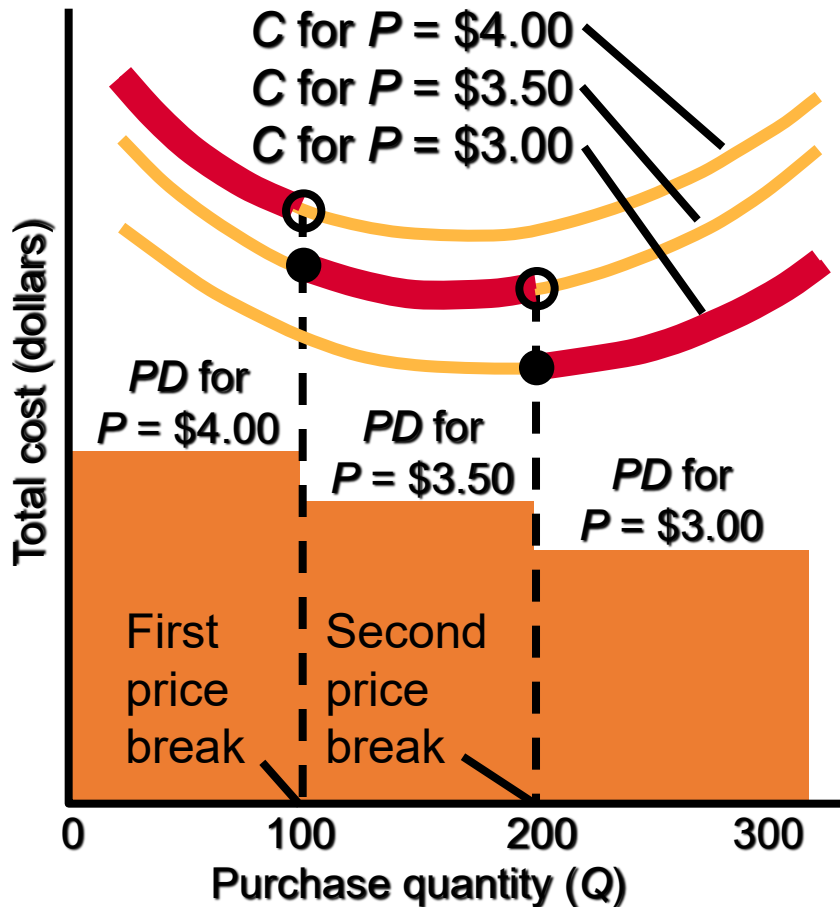
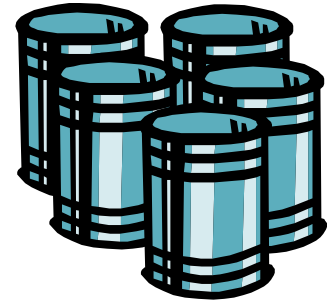


(b) EOQs and price break quantities

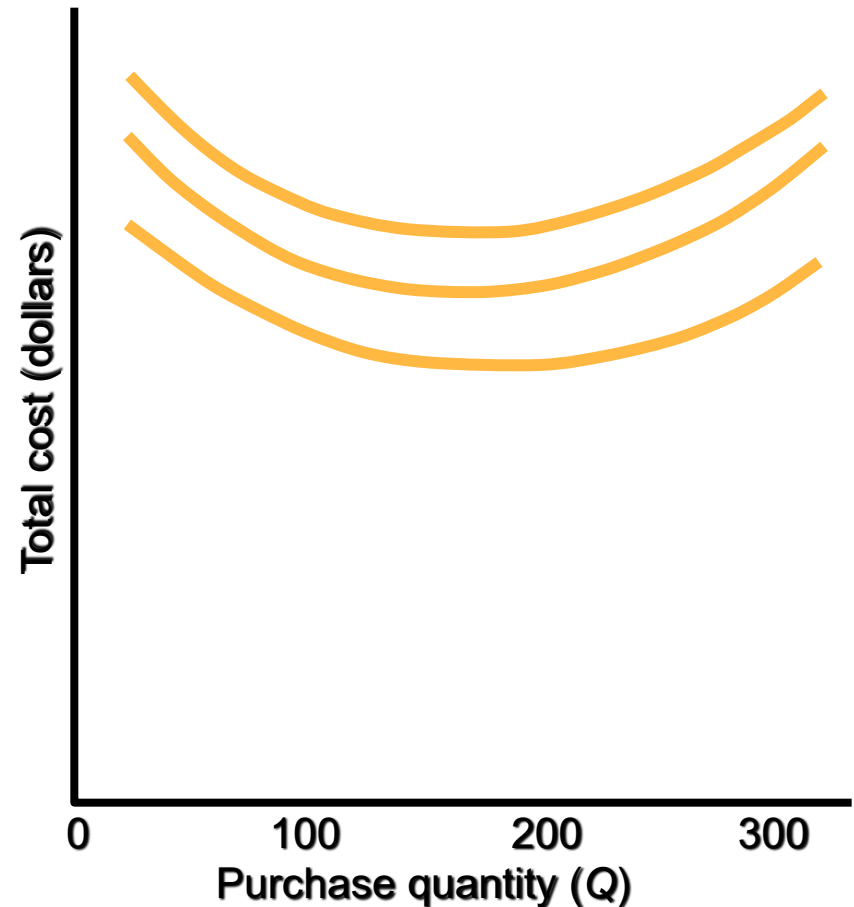
Special Inventory Models

Quantity Discounts

Figure E.3



(a) Total cost curves with purchased materials added



(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

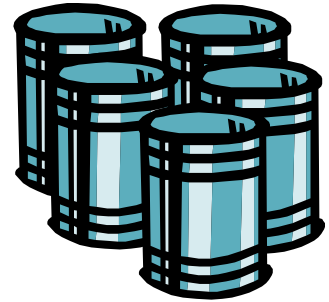
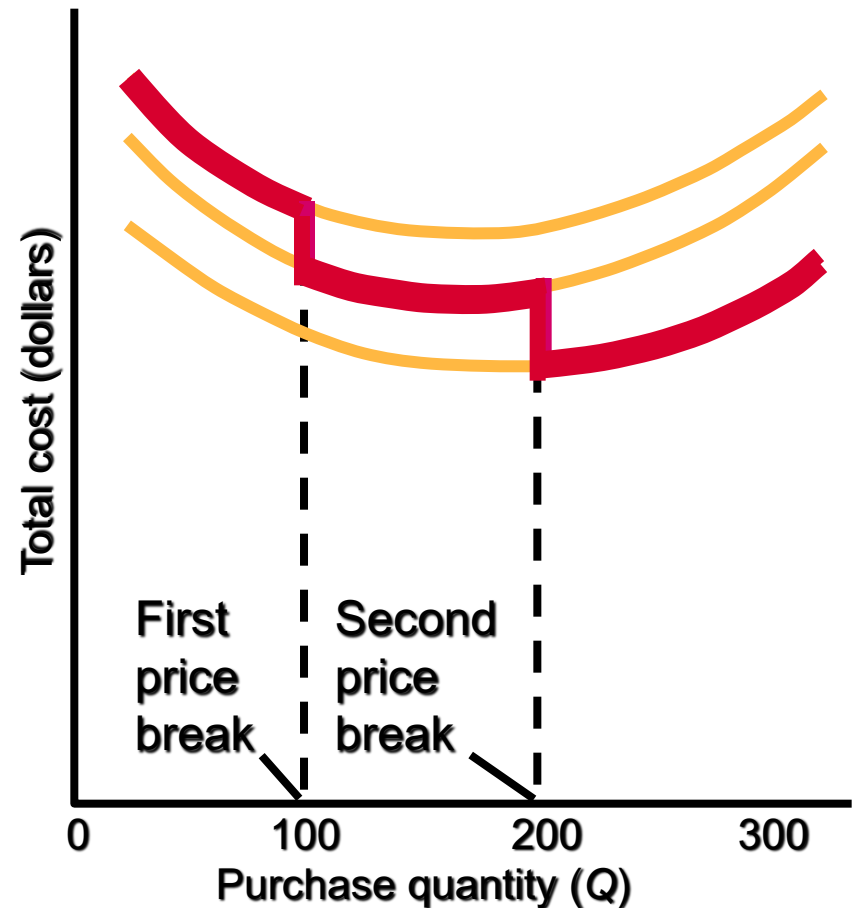
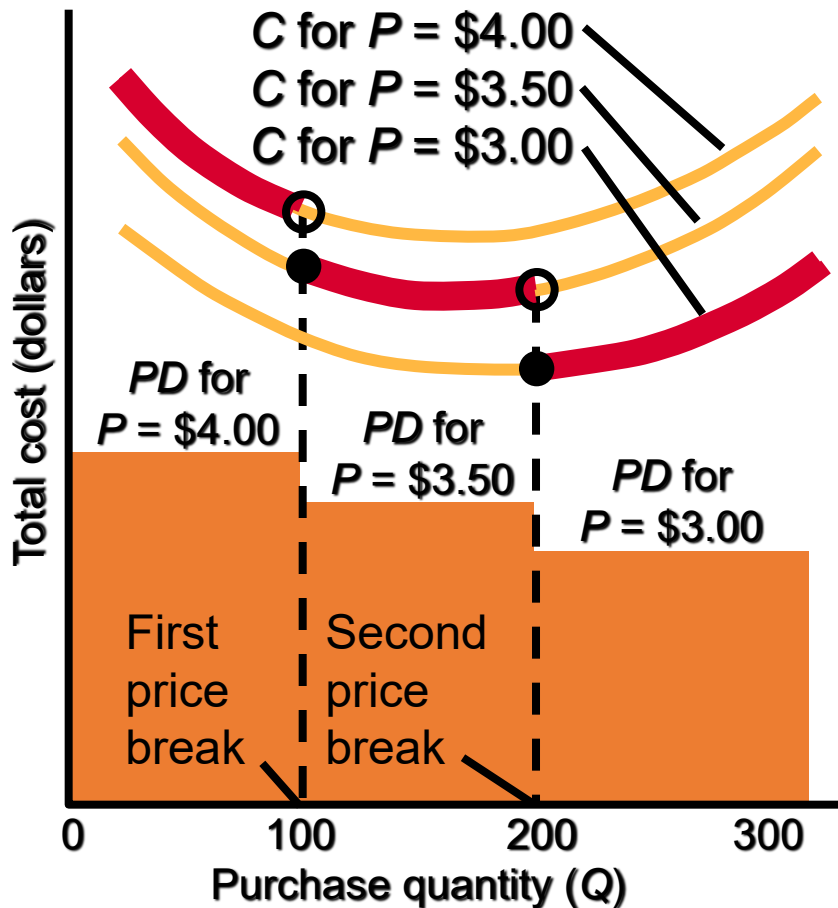


Figure E.3



(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

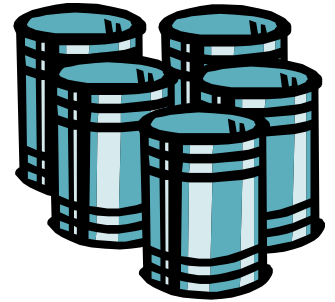
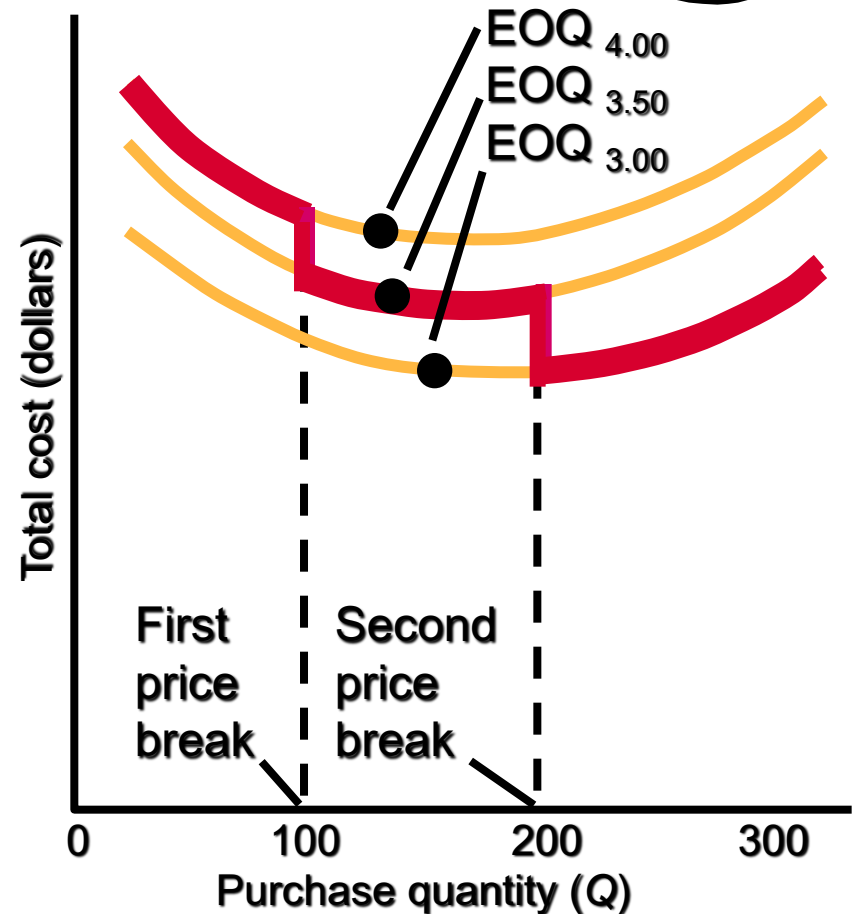
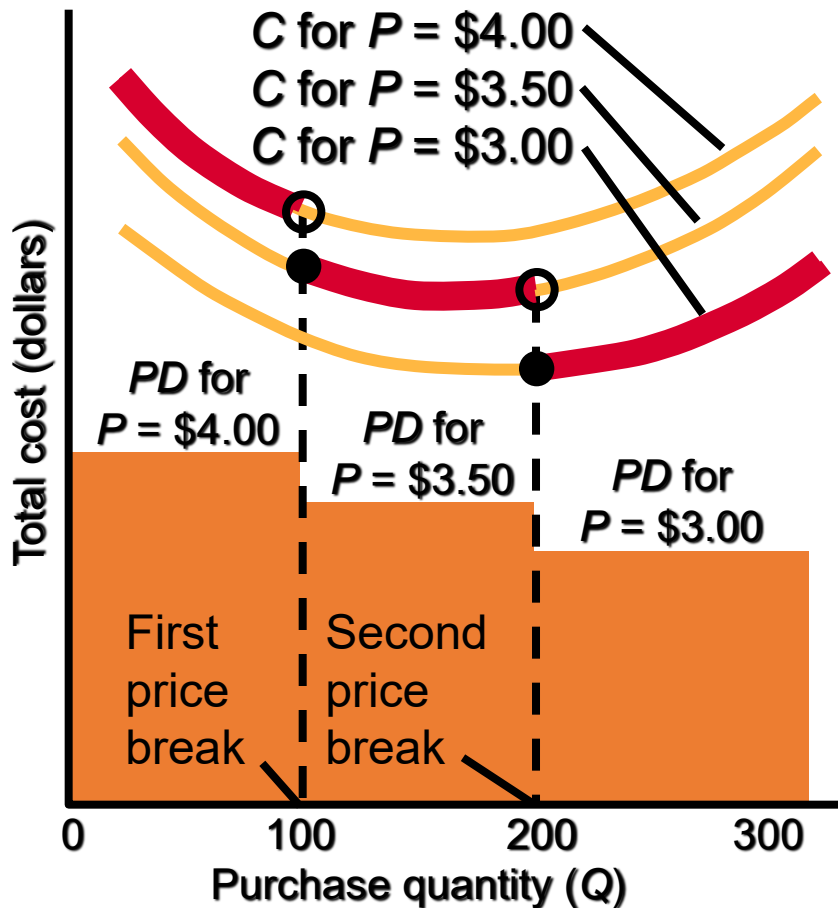


Figure E.3



(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts



Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2DS}{H}}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = 77 \text{ units}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

$EOQ_{57.00} = 77 \text{ units}$

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ = 77 units~~

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

EOQ_{\$58.80} = 76 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

EOQ_{\$58.80} = 76 units

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S) + PD$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \frac{75}{2} [(0.25)(\$60.00)] + \frac{936}{75} (\$45) + \$60.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \frac{300}{2} [(0.25)(\$58.80)] + \frac{936}{300} (\$45) + \$58.80(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \frac{500}{2} [(0.25)(\$57.00)] + \frac{936}{500} (\$45) + \$57.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \$56,999$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \$56,999$$

Example E.2

Discount

Whole Nature Foods sells a gluten-free product for which the annual demand is 5,000 boxes. At the moment, it is paying \$6.40 for each box; carrying cost is 25% of the unit cost; ordering costs are \$25. A new supplier has offered to sell the same item for \$6.00 if Whole Nature Foods buys at least 3,000 boxes per order. Should the firm stick with the old supplier, or take advantage of the new quantity discount?

Special Inventory Models

One-Period Decisions



Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10					
20					
30					
40					
50					

For $Q = D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10					
20					
30					
40					
50					

For $Q = D$
Payoff = $(\$10)(10)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100				
20					
30					
40					
50					

For $Q = D$
Payoff = \$100

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20					
30					
40					
50					

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$

$$\text{Payoff} = pD - l(Q - D)$$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$
 Payoff = $(\$10)(30)$
 $- (\$5)(40 - 30)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$

Payoff = $(\$10)(30)$

– $(\$5)(40 - 30)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$
Payoff = \$250

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40			250	400	400
50					500

For $Q > D$
Payoff = \$250

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

For $Q > D$

$$\text{Payoff} = pD - l(Q - D)$$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ =

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = 0.2(\$0)

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = $0.2(\$0) + 0.3(\$150)$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = 0.2(\$0) + 0.3(\$150) + 0.3(\$300)

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

$$\text{Expected payoff}_{30} = 0.2(\$0) + 0.3(\$150) + 0.3(\$300) + 0.1(\$300)$$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

$$\text{Expected payoff}_{30} = 0.2(\$0) + 0.3(\$150) + 0.3(\$300) + 0.1(\$300) + 0.1(\$300)$$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = \$195

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

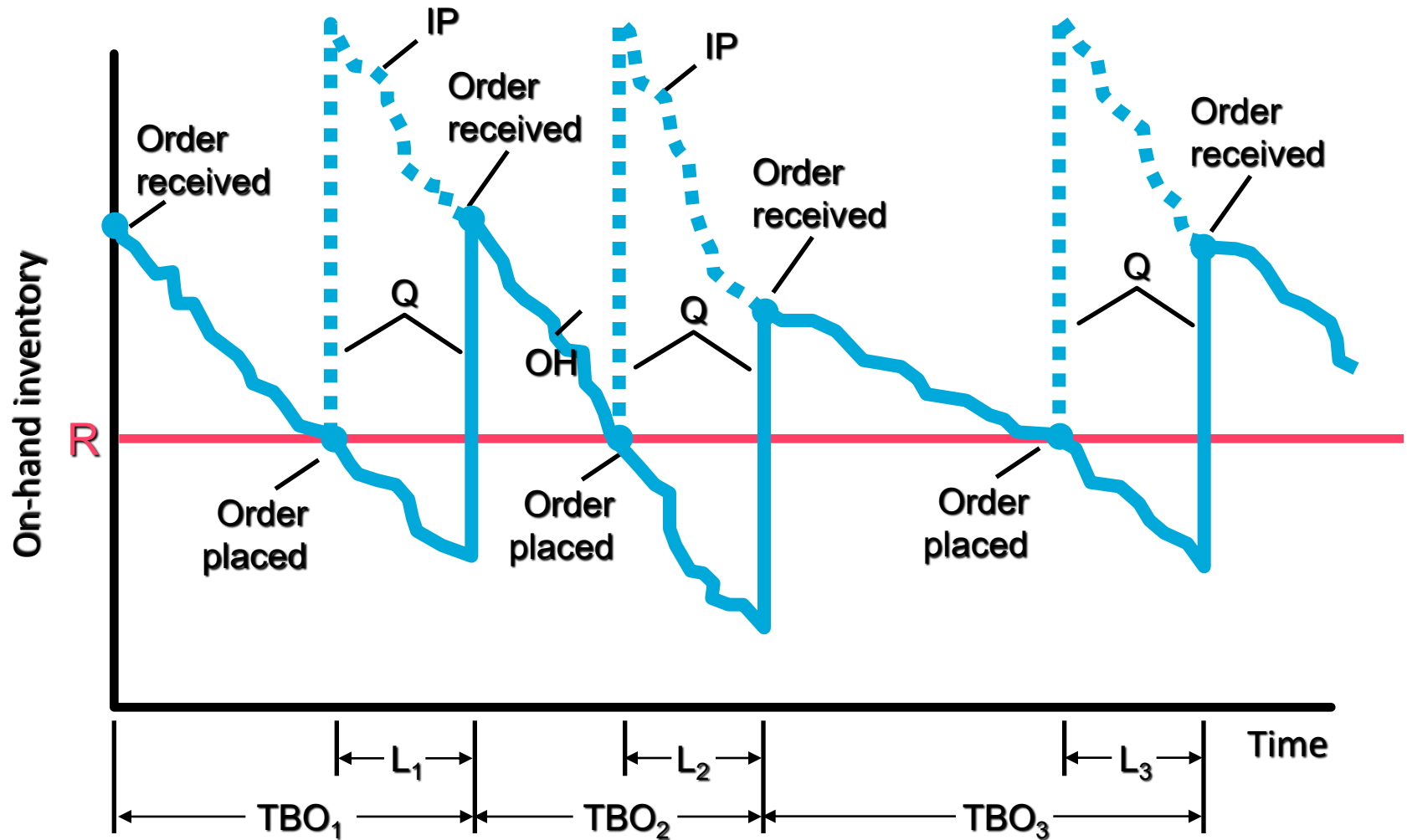
Expected payoff₃₀ = \$195

Q	<i>D</i>					Expected Payoff
	10	20	30	40	50	
10	\$100	\$100	\$100	\$100	\$100	
20	50	200	200	200	200	
30	0	150	300	300	300	195
40	-50	100	250	400	400	
50	-100	50	200	350	500	

Example E.3

Uncertain Demand

Figure 15.8



Reorder Point / Safety Stock

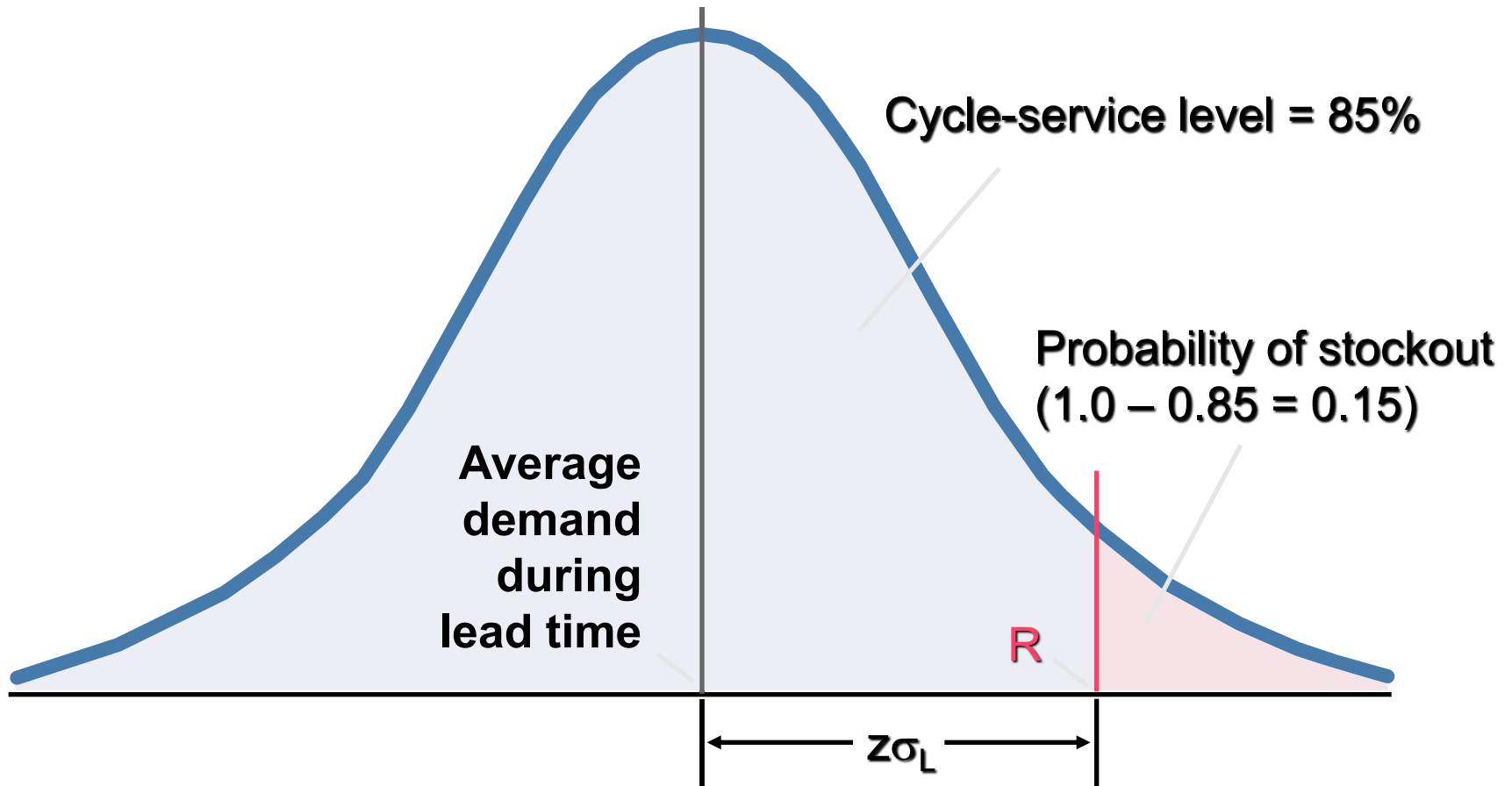


Figure 15.9

EXAMPLE

Records show that the demand for dishwasher detergent during the lead time is normally distributed, with an average of 250 boxes and *variance* $l = 22$.

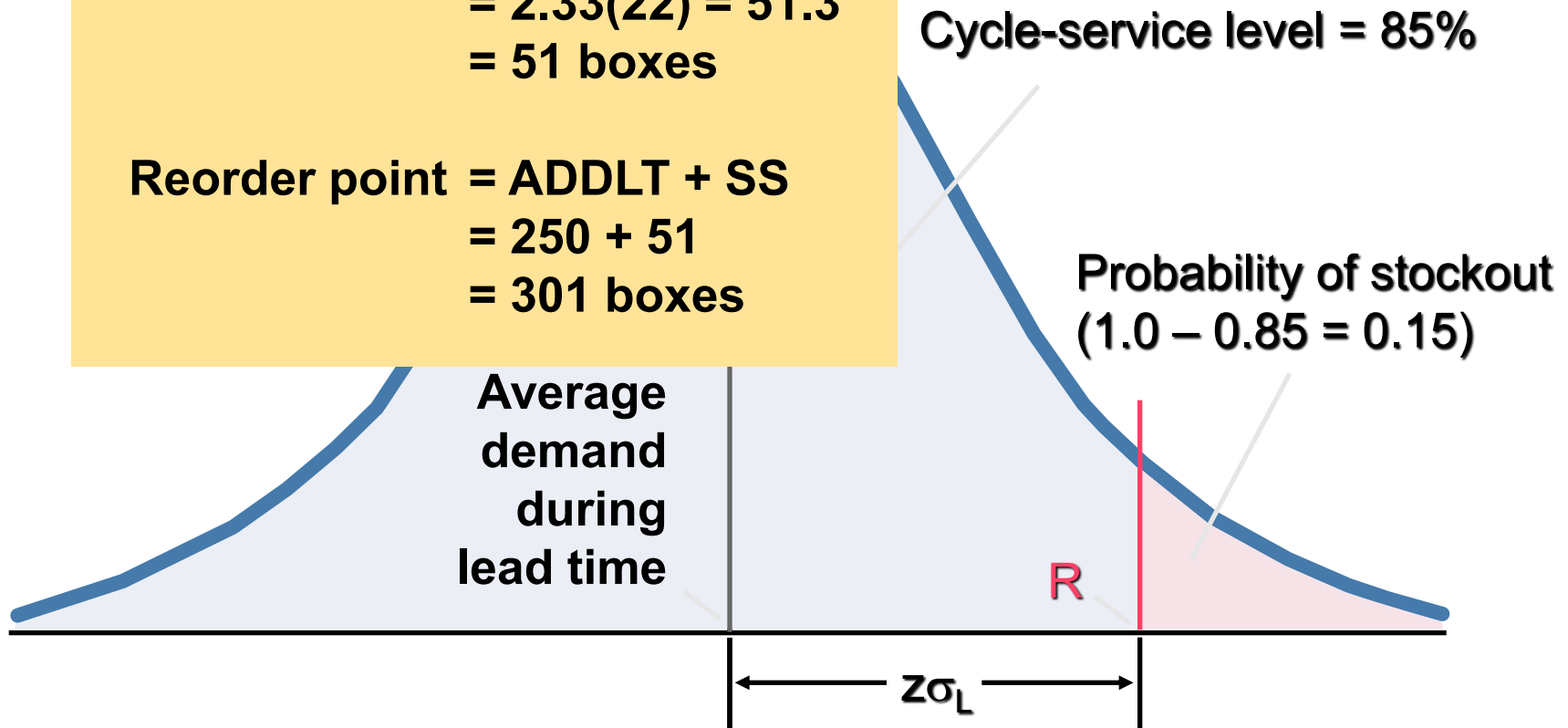
What safety stock should be carried for a 99 percent cycle-service level? What is R ?

Reorder Point / Safety Stock

Safety Stock/R

$$\begin{aligned}\text{Safety stock} &= z\sigma_L \\ &= 2.33(22) = 51.3 \\ &= 51 \text{ boxes}\end{aligned}$$

$$\begin{aligned}\text{Reorder point} &= \text{ADDLT} + \text{SS} \\ &= 250 + 51 \\ &= 301 \text{ boxes}\end{aligned}$$



Lead Time Distributions

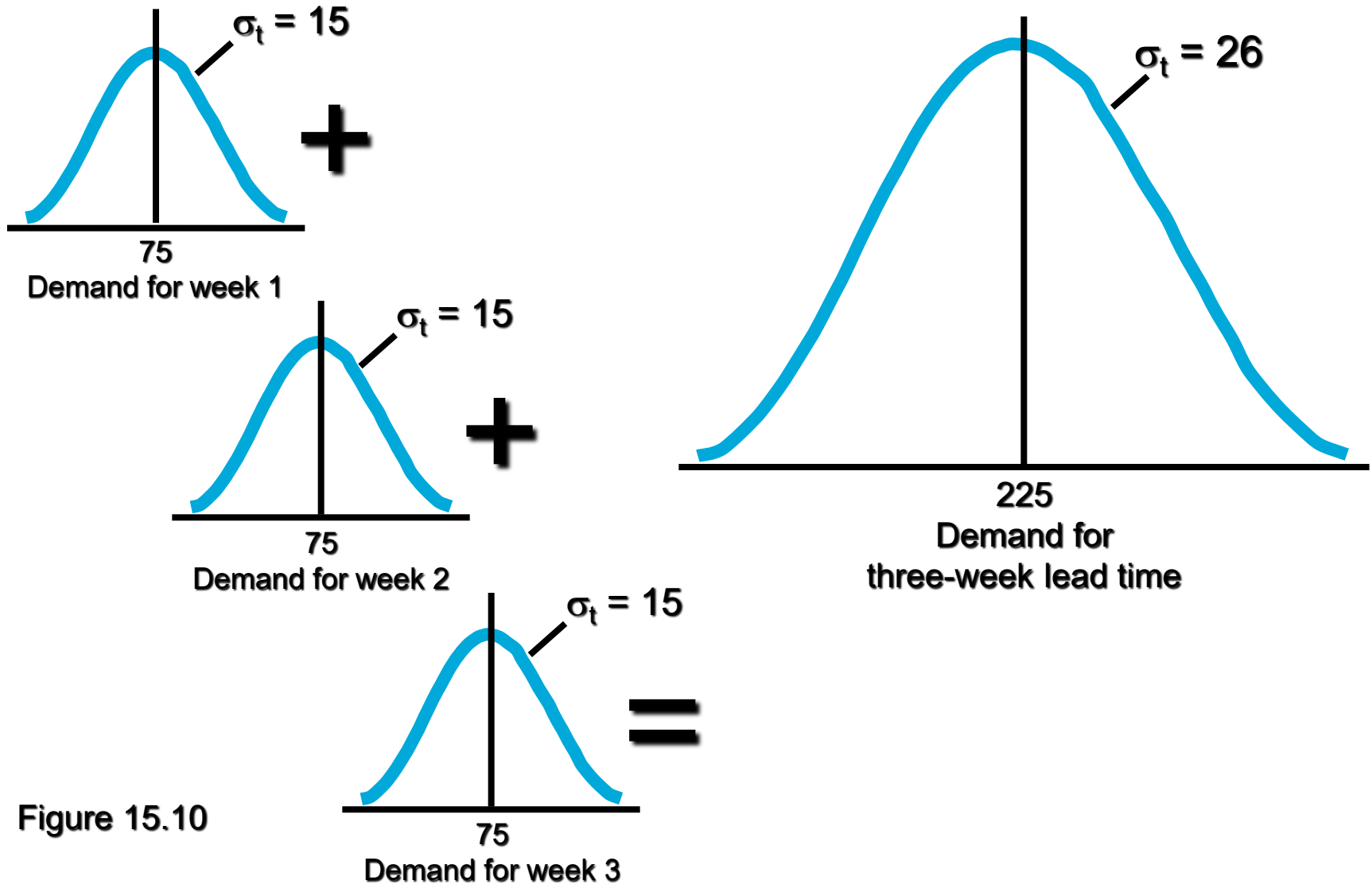
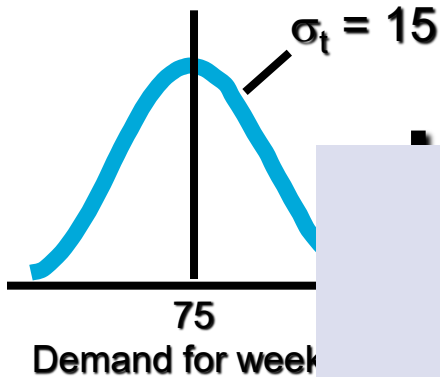


Figure 15.10

Example Finding the safety stock and R When the Demand Distribution for Lead Time must Be Developed

- Let us return to the bird feeder example. Suppose that the average demand is 18 units per week \
- with a standard deviation of 5 units. The lead time is constant at two weeks. Determine the safety stock and reorder point if management wants a 90 percent cycle-service level. What is the j total cost of the Q system?

Lead Time Distributions



Bird feeder Lead Time Distribution

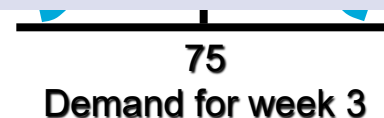
$$t = 1 \text{ week} \quad d = 18 \quad L = 2$$

$$\sigma_L = \sigma_t \sqrt{L} = 5\sqrt{2} = 7.1$$

$$\text{Safety stock} = z\sigma_L = 1.28(7.1) = 9.1 \text{ or } 9 \text{ units}$$

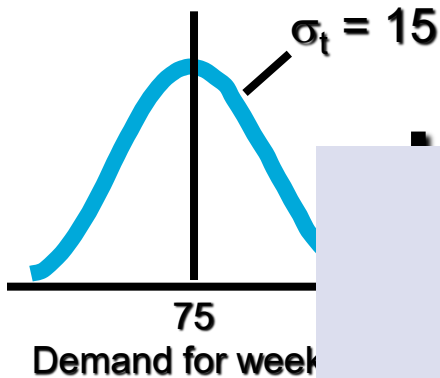
$$\begin{aligned} \text{Reorder point} &= dL + \text{Safety stock} \\ &= 2(18) + 9 = 45 \text{ units} \end{aligned}$$

Example 15.6



[illegible]

Lead Time Distributions



Bird feeder Lead Time Distribution

$$t = 1 \text{ week} \quad d = 18 \quad L = 2$$

$$\text{Reorder point} = 2(18) + 9 = 45 \text{ units}$$

$$C = \frac{75}{2}(\$15) + \frac{936}{75}(\$45) + 9(\$15)$$

$$C = \$562.50 + \$561.60 + \$135 = \$1259.10$$

Example 15.6



Lead Time Distributions

TABLE 15.1 PROBABILITY DISTRIBUTION FOR LEAD TIME

Lead Time (weeks)	Probability for Lead Time
1	0.35
2	0.45
3	0.10
4	0.05
5	0.05

TABLE 15.2 PROBABILITY DISTRIBUTION FOR DEMAND

Demand (units per week)	Probability of Demand
10	0.10
13	0.20
18	0.40
23	0.20
26	0.10

Other Probabilistic Models (1 of 4)

- When data on demand during lead time are not available, there are other models available
 1. When demand is variable and lead time is constant
 2. When lead time is variable and demand is constant
 3. When both demand and lead time are variable

Other Probabilistic Models (2 of 4)

Demand is variable and lead time is constant

$$\text{ROP} = (\text{Average daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT}$$

where $\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$

σ_d = Standard deviation of demand per day

Probabilistic Example

Average daily demand (normally distributed) = 15

Lead time in days (constant) = 2

Standard deviation of daily demand = 5

Service level = 90%

Z for 90% = 1.28

From Appendix I

$$\begin{aligned}\text{ROP} &= (15 \text{ units} \times 2 \text{ days}) + Z\sigma_{dLT} \\ &= 30 + 1.28(5)(\sqrt{2}) \\ &= 30 + 9.02 = 39.02 \approx 39\end{aligned}$$

Safety stock is about 9 computers

Gauss table and random distribution Appendix

[illegible]

Other Probabilistic Models (3 of 4)

Lead time is variable and demand is constant

$$\text{ROP} = (\text{Daily demand} \times \text{Average lead time in days}) + Z \times (\text{Daily demand}) \times \sigma_{LT}$$

where σ_{LT} = Standard deviation of lead time in days

Probabilistic Example (3 of 3)

Daily demand (constant) = 10

Average lead time = 6 days

Standard deviation of lead time = $\sigma_{LT} = 1$

Service level = 98%, so Z (from Appendix I) = 2.055

$$\begin{aligned}\text{ROP} &= (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1) \\ &= 60 + 20.55 = 80.55\end{aligned}$$

Reorder point is about 81 cameras

Other Probabilistic Models (4 of 4)

Both demand and lead time are variable

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time}) + Z\sigma_{LT}$$

where σ_d = Standard deviation of demand per day

e σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma_{LT}^2}$$

Probabilistic Example

Average daily demand (normally distributed) = 150

Standard deviation = $\sigma_d = 16$

Average lead time 5 days (normally distributed)

Standard deviation = $\sigma_{LT} = 1$ day

Service level = 95%, so $Z = 1.645$ (from Appendix I)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.645\sigma_{dLT}$$

$$\begin{aligned}\sigma_{dLT} &= \sqrt{(5 \text{ days} \times 16^2) + (150^2 \times 1^2)} = \sqrt{(5 \times 256) + (22,500 \times 1)} \\ &= \sqrt{(1,280) + (22,500)} = \sqrt{23,780} \cong 154\end{aligned}$$

$$\text{ROP} = (150 \times 5) + 1.645(154) \cong 750 + 253 = 1,003 \text{ packs}$$

Single-Period Model

- Only *one* order is placed for a product
- Units have little or no value at the end of the sales period

C_s = Cost of shortage = Sales price/unit – Cost/unit

C_o = Cost of overage = Cost/unit – Salvage value

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single-Period Example (1 of 2)

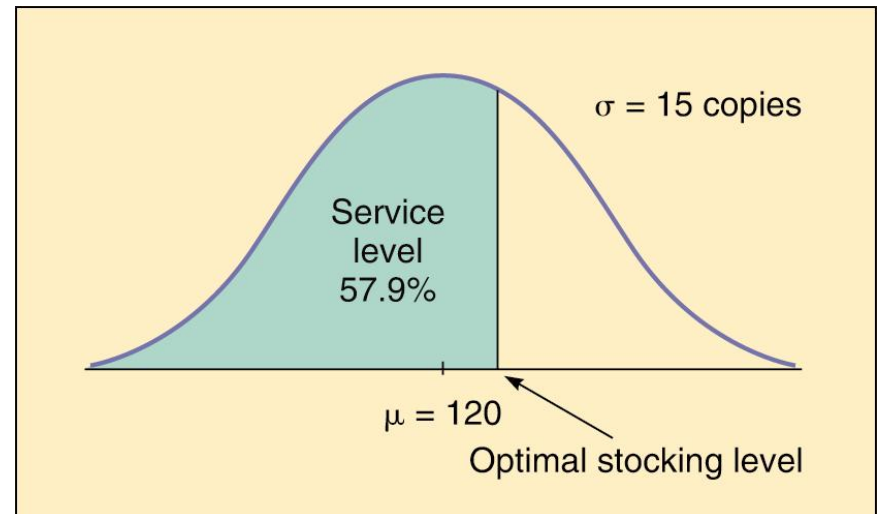
Average demand = $\mu = 120$ papers/day

Standard deviation = $\sigma = 15$ papers

C_s = cost of shortage = $\$1.25 - \$0.70 = \$0.55$

C_o = cost of overage = $\$0.70 - \$0.30 = \$0.40$

$$\begin{aligned}\text{Service level} &= \frac{C_s}{C_s + C_o} \\ &= \frac{.55}{.55 + .40} \\ &= \frac{.55}{.95} = .579\end{aligned}$$



Single-Period Example (2 of 2)

From Appendix I, for the area .579, $Z \cong .199$

The optimal stocking level

$$= 120 \text{ copies} + (.199)(\sigma)$$

$$= 120 + (.199)(15) = 120 + 3 = 123 \text{ papers}$$

The stockout risk = $1 - \text{Service level}$

$$= 1 - .579 = .421 = 42.1\%$$

OTHER QUESTION

- The demand at Arnold Palmer Hospital for a specialized surgery pack is 60 per week, virtually every week. The lead time from McKesson, its main supplier, is normally distributed, with a mean of 6 weeks for this product and a standard deviation of 2 weeks. A 90% weekly service level is desired. Find the ROP.

Safety stock

- The inclusion of safety stock (ss) changed the expression to
 - $ROP = d * L + ss$
 - The amount of safety stock maintained depends on the cost of incurring a stockout and the cost of holding the extra inventory. Annual stockout cost is computed as follows:
- Annual stockout costs =
 - The sum of the units short for each demand level * The probability of that demand level * The stockout cost / unit * The number of orders per year

Periodic Review Systems

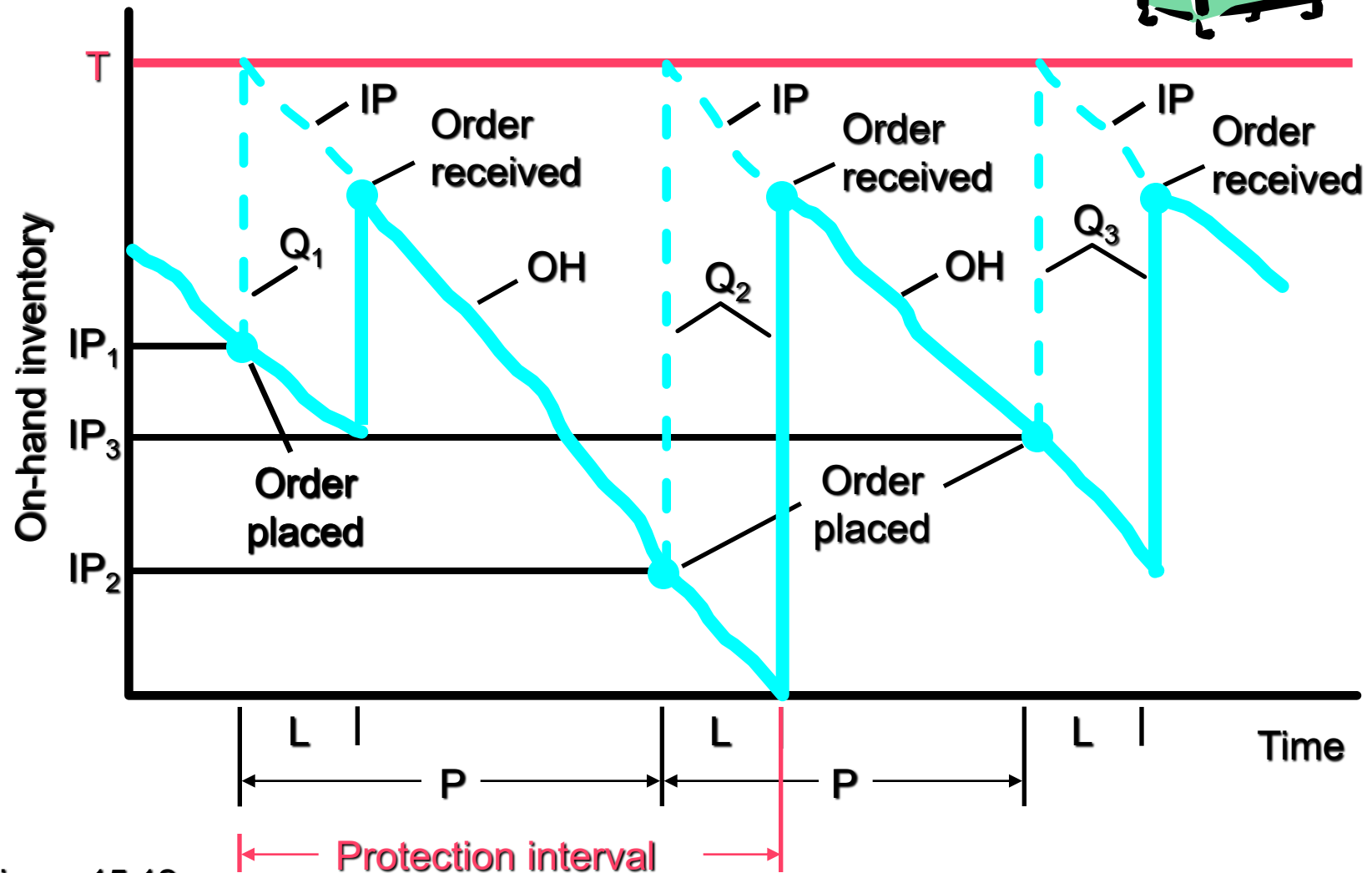
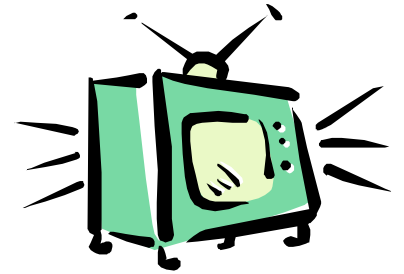


Figure 15.12

Example Calculating P and T

- Again, let us return to the bird feeder example. Recall that demand for the bird feeder is normally distributed with a mean of 18 units per week and a standard deviation in weekly demand of 5 units. The lead time is 2 weeks, and the business operates 52 weeks per year. The Q system developed in Example 15.6 called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent. What is the equivalent *P* system? *What is the total cost ? Answers are to be rounded to the nearest integer.*

Periodic Review Systems



Bird feeder— Calculating P and T

$\sigma_t = 5$ units $L = 2$ weeks cycle/service level = 90%

EOQ = 75 units $D = (18 \text{ units/week})(52 \text{ weeks}) = 936$ units

$$P = \frac{\text{EOQ}}{D} (52) = \frac{75}{936} (52) = 4.2 \text{ or } 4 \text{ weeks}$$

$$\sigma_{P+L} = \sigma_t \sqrt{P + L} = 5 \sqrt{6} = 12 \text{ units}$$

T = Average demand during the protection interval + Safety stock
= $d(P + L) + z\sigma_{P+L}$
= $(18 \text{ units/week})(6 \text{ weeks}) + 1.28(12 \text{ units}) = 123 \text{ units}$

Example 15.8

← Protection interval →

Periodic Review Systems



Bird feeder— Calculating P and T

$\sigma_t = 18$ units $L = 2$ weeks cycle/service level = 90%

EOQ = 75 units $D = (18 \text{ units/week})(52 \text{ weeks}) = 936$ units

$P = 4$ weeks $T = 123$ units

$$C = \frac{4(18)}{2} (\$15) + \frac{936}{4(18)} (\$45) + 15(\$15)$$

$$C = \$540 + \$585 + \$225 = \$1350$$

On-hand inventory

← Protection interval →

Comparison of Q and P Systems

P Systems

- Convenient to administer
- Orders may be combined
- IP only required at review

Q Systems

- Individual review frequencies
- Possible quantity discounts
- Lower, less-expensive safety stocks



QUIZ

INVENTORIES

1-ABC analysis divides on-hand inventory into three classes, based on:

- a) unit price.
- b) the number of units on hand.
- c) annual demand.
- d) annual dollar values.

2-Cycle counting:

- a) provides a measure of inventory turnover.
- b) assumes that all inventory records must be verified with the same frequency.
- c) is a process by which inventory records are periodically verified.
- d) all of the above.

3-The two most important inventory-based questions answered by the typical inventory model are:

- a) when to place an order and the cost of the order.
- b) when to place an order and how much of an item to order.
- c) how much of an item to order and the cost of the order.
- d) how much of an item to order and with whom the order should be placed.

4- Extra units in inventory to help reduce stockouts are called:

- a) reorder point.
- b) safety stock.
- c) just-in-time inventory.
- d) all of the above

5-The difference(s) between the basic EOQ model and the production order quantity model is(are) that:

- a) the production order quantity model does not require the assumption of known, constant demand.
- b) the EOQ model does not require the assumption of negligible lead time.
- c) the production order quantity model does not require the assumption of instantaneous delivery.
- d) all of the above.

6-The EOQ model with quantity discounts attempts to determine:

- a) the lowest amount of inventory necessary to satisfy a certain service level.
- b) the lowest purchase price.
- c) whether to use a fixed-quantity or fixed-period order policy.
- d) how many units should be ordered.
- e) the shortest lead time.

7-The appropriate level of safety stock is typically determined by:

- a) minimizing an expected stockout cost.
- b) choosing the level of safety stock that assures a given service level.
- c) carrying sufficient safety stock so as to eliminate all stockouts.
- d) annual demand

Problem 1

Booker's Book Bindery divides inventory items into three classes, according to their dollar usage. Calculate the usage values of the following inventory items and determine which is most likely to be classified as an A item.

PART NUMBER	DESCRIPTION	QUANTITY USED PER YEAR	UNIT VALUE (\$)
1	Boxes	500	3.00
2	Cardboard (square feet)	18,000	0.02
3	Cover stock	10,000	0.75
4	Glue (gallons)	75	40.00
5	Inside covers	20,000	0.05
6	Reinforcing tape (meters)	3,000	0.15
7	Signatures	150,000	0.45

Problem 2

A regional warehouse purchases hand tools from various suppliers and then distributes them on demand to retailers in the region. The warehouse operates five days per week, 52 weeks per year. Only when it is open can orders be received. The following data are estimated for 3/8-inch hand drills with double insulation and variable speeds:

Average daily demand = 100 drills

Standard deviation of daily demand (σ_d) = 30 drills

Lead time (L) = 3 days

Holding cost (H) = \$9.40/unit/year

Ordering cost (S) = \$35/order

Cycle-service level = 92 percent

The warehouse uses a continuous review (Q) system.

- a. What order quantity, Q and reorder point, R , should be used?
- b. If on-hand inventory is 40 units, there is one open order for 440 drills, and there are no backorders, should a new order be placed?

[illegible]

Problem 3

Suppose that a periodic review (P) system is used at the warehouse, but otherwise the data are the same as in Solved Problem 5.

- a. Calculate the P (in workdays, rounded to the nearest day) that gives approximately the same number of orders per year as the EOQ.
- b. What is the value of the target inventory level, T ? Compare the P system to the Q system in Solved Problem 5.
- c. It is time to review the item. On-hand inventory is 40 drills; there is a scheduled receipt of 440 drills and no backorders. How much should be reordered?

David Rivera Optical has determined that its reorder point for eyeglass frames is 50 ($d \times L$) units. Its carrying cost per frame per year is \$5, and stockout (or lost sale) cost is \$40 per frame. The store has experienced the following probability distribution for inventory demand during the lead time (reorder period). The optimum number of orders per year is six.

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP \rightarrow 50	.3
60	.2
70	.1
	<u>1.0</u>

How much safety stock should David Rivera keep on hand?

APPROACH ► The objective is to find the amount of safety stock that minimizes the sum of the additional inventory holding costs and stockout costs. The annual holding cost is simply the holding cost per unit multiplied by the units added to the ROP. For example, a safety stock of 20 frames, which implies that the new ROP, with safety stock, is 70 ($= 50 + 20$), raises the annual carrying cost by $\$5(20) = \100 .

However, computing annual stockout cost is more interesting. For any level of safety stock, stockout cost is the expected cost of stocking out. We can compute it, as in Equation (12-12), by multiplying the number of frames short (Demand $-$ ROP) by the probability of demand at that level, by the stockout cost, by the number of times per year the stockout can occur (which in our case is the number of orders per year). Then we add stockout costs for each possible stockout level for a given ROP.⁴

SOLUTION ► We begin by looking at zero safety stock. For this safety stock, a shortage of 10 frames will occur if demand is 60, and a shortage of 20 frames will occur if the demand is 70. Thus the stockout costs for zero safety stock are:

$$\begin{aligned} & (10 \text{ frames short}) (.2) (\$40 \text{ per stockout}) (6 \text{ possible stockouts per year}) \\ & + (20 \text{ frames short}) (.1) (\$40)(6) = \$960 \end{aligned}$$

The following table summarizes the total costs for each of the three alternatives:

SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	$(20)(\$5) = \100	\$ 0	\$100
10	$(10)(\$5) = \$ 50$	$(10) (.1) (\$40) (6) = \240	\$290
0	\$ 0	$(10) (.2) (\$40) (6) + (20) (.1) (\$40) (6) = \$960$	\$960

The safety stock with the lowest total cost is 20 frames. Therefore, this safety stock changes the reorder point to $50 + 20 = 70$ frames.

INSIGHT ► The optical company now knows that a safety stock of 20 frames will be the most economical decision.



Q System

$$\text{ROP} = d(\text{daily}) \times \text{Lead time} + \text{safety stock}$$
$$(\text{= } z \cdot \text{Standard deviation } L)$$

P System

$$\text{ROP} = d \text{ daily} \times (\text{Period} + \text{Lead time}) + \text{safety stock } (p+l)$$

$$\text{Period} = \frac{\text{EOQ}}{\text{Annual demand}}$$