

$$\Omega \cong \diamond\mathcal{W}$$

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## 1 Definition

### 1.1 $\Omega$

There is a correct process  $c$  such that **eventually**  $\Omega_p = c$  **forever** for every correct process  $p$ .

### 1.2 $\diamond\mathcal{W}$

Eventual Weak Accuracy + Weak Completeness :

**Eventual weak accuracy:** *There is a time after which some correct process is never suspected by any correct process.*

**Weak Completeness:** Every faulty process is eventually permanently suspected by *some* correct process.

## 2 System Assumptions

1. Complete network topology.
2. Asynchronous identified processes: a process and its identifier are used equivalently.
3. Asynchronous reliable links (not necessarily FIFO).

We consider executions as sequences of (atomic) events  $(e_i)_{i \geq 0}$  where each event  $e_i$  occurs at a given process during the time interval from time  $i$  to time  $i + 1$ . An event may be:

- an internal event, corresponding to a local computation performed by the process;
- a send event, in which the process sends a message;
- or a receive event, in which the process receives a message.

## 3 Notations

- $V$ : the set of processes
- $Correct \subseteq V$ : the set of correct processes.
- $Faulty \subseteq V$ : the set of faulty processes.
- $X_p$ : the value of variable  $X$  of process  $p$ .
- $X_p^t$ : the value of variable  $X$  of process  $p$  at time  $t$ .

## 4 $T_{\Omega \rightarrow \diamond \mathcal{W}}$

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**Algorithm 1**  $T_{\Omega \rightarrow \diamond \mathcal{W}}$ , code for every process  $p$

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1: Function  $T_{\Omega \rightarrow \diamond \mathcal{W}}(p)$   
2:   return  $V \setminus \{\Omega_p\}$   
3: End Function
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**Question 1.** Prove that  $T_{\Omega \rightarrow \diamond \mathcal{W}}$  satisfies weak completeness.

**Question 2.** Prove that  $T_{\Omega \rightarrow \diamond \mathcal{W}}$  satisfies eventual weak accuracy.

## 5 $T_{\diamond \mathcal{W} \rightarrow \Omega}$

**Question 1.** Following the principles presented in the lesson, propose an algorithm  $T_{\diamond \mathcal{W} \rightarrow \Omega}$ .

*Notations:* Please use the following variables:

- $Leader \in V$ , initialized to  $\min V$ .
- $C[]$ , array of integers indexed on  $V$ , every cell is initialized to 0.

**Question 2.** Show the following lemma.

**Lemma 1.**  $\forall p \in \text{Correct}, \forall q \in \text{Faulty}, \forall t \in \mathbb{N}, \exists t' > t$  such that  $C[q]_p^t < C[q]_p^{t'}$ .

**Question 3.** Show the following lemma.

**Lemma 2.**  $\exists p \in \text{Correct}$  and  $\exists k, t \in \mathbb{N}$  such that  $\forall q \in \text{Correct}, \forall t' \geq t, C[p]_q^{t'} \leq k$ .

**Question 4.** Show the following lemma.

**Lemma 3.**  $\exists t \in \mathbb{N}$  such that  $\forall p \in \text{Correct}, \forall t' \geq t, C_p^t[\text{Leader}_p^t] = C_p^{t'}[\text{Leader}_p^{t'}]$ .

**Question 5.** Show the following corollary.

**Corollary 1.**  $\exists t \in \mathbb{N}$  such that  $\forall p \in \text{Correct}, \forall t' \geq t, \text{Leader}_p^t = \text{Leader}_p^{t'}$ .

**Question 6.** Show the following lemma.

**Lemma 4.**  $\exists \ell \in \text{Correct}$  and  $\exists t \in \mathbb{N}$  such that  $\forall t' \geq t, \forall p \in \text{Correct}, \text{Leader}_p^{t'} = \ell$ .