$\Omega \cong \diamond\mathcal{W}$

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1 Definition

1.1 Ω

Eventually every correct process p satisfies $\Omega_p = c$ **forever**, where c is a correct process.

1.2 $\diamond\mathcal{W}$

Eventual Weak Accuracy + Weak Completeness :

Eventual weak accuracy: *There is a time after which some* correct process is never suspected by any correct process.

Weak Completeness: Every faulty process is eventually permanently suspected by *some* correct process.

2 System Assumptions

1. Complete network topology.
2. Asynchronous identified processes: a process and its identifier are used equivalently.
3. Asynchronous reliable links (not necessarily FIFO).

We consider executions as sequences of (atomic) events $(e_i)_{i\geq 0}$ where each event e_i occurs at a given process during the time interval from time i to time $i+1$. An event may be:

- an internal event, corresponding to a local computation performed by the process;
- a send event, in which the process sends a message;
- or a receive event, in which the process receives a message.

3 Notations

- V: the set of processes
- $Correct \subseteq V$: the set of correct processes.
- $Faulty \subseteq V$: the set of faulty processes.
- X_p: the value of variable X of process p.
- X_p^t: the value of variable X of process p at time t.

4 $T_{\Omega \to \diamond \mathcal{W}}$

Algorithm 1 $T_{\Omega \to \diamond \mathcal{W}}$, code for every process p

```
Function T_{Ω→◇W}(p)
    return V \ {Ω_p}
End Function
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Question 1. Prove that $T_{\Omega \to \diamond \mathcal{W}}$ satisfies weak completeness.

Question 2. Prove that $T_{\Omega \to \diamond \mathcal{W}}$ satisfies eventual weak accuracy.

5 $T_{\diamond\mathcal{W}\to\Omega}$

Question 1. Following the principles presented in the lesson, propose an algorithm $T_{\diamond\mathcal{W}\to\Omega}$.

Notations: Please use the following variables:

- $Leader \in V$, initialized to $\min V$.
- $C[]$, array of integers indexed on V, every cell is initialized to 0.

Question 2. Show the following lemma.

Lemma 1. *$\forall p \in Correct$, $\forall q \in Faulty$, $\forall t \in \mathbb{N}$, $\exists t' > t$ such that $C[q]_p^t < C[q]_p^{t'}$.*

Question 3. Show the following lemma.

Lemma 2. *If $Correct \neq \emptyset$, then $\exists p \in Correct$ and $\exists k, t \in \mathbb{N}$ such that $\forall q \in Correct$, $\forall t' \geq t$, $C[p]_q^{t'} \leq k$.*

Question 4. Show the following lemma.

Lemma 3. $\exists t \in \mathbb{N}$ *such that* $\forall p \in Correct,\ \forall t' \geq t,\ C_p^t[Leader_p^t] = C_p^{t'}[Leader_p^{t'}]$.

Question 5. Show the following corollary.

Corollary 1. $\exists t \in \mathbb{N}$ *such that* $\forall p \in Correct,\ \forall t' \geq t,\ Leader_p^t = Leader_p^{t'}$.

Question 6. Show the following lemma.

Lemma 4. $\exists t \in \mathbb{N}$ *such that* $\forall t' \geq t,\ \forall p \in Correct,\ Leader_p^{t'} = \ell$ *where* $\ell \in Correct$.