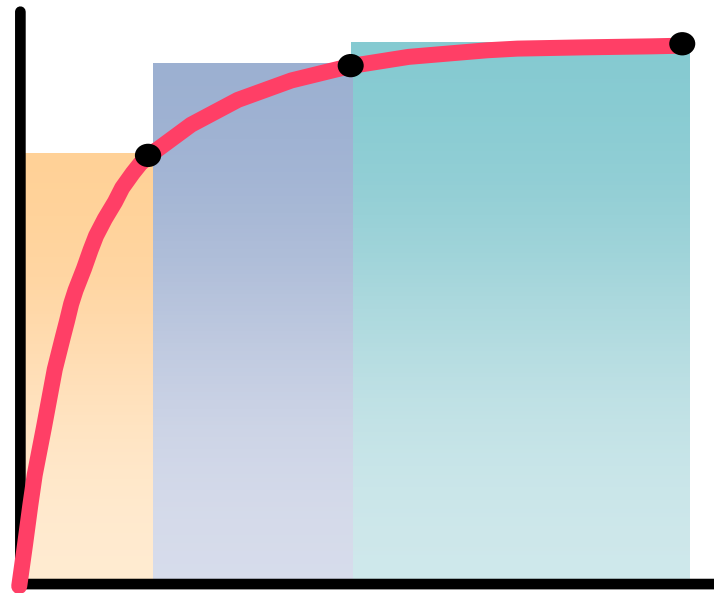


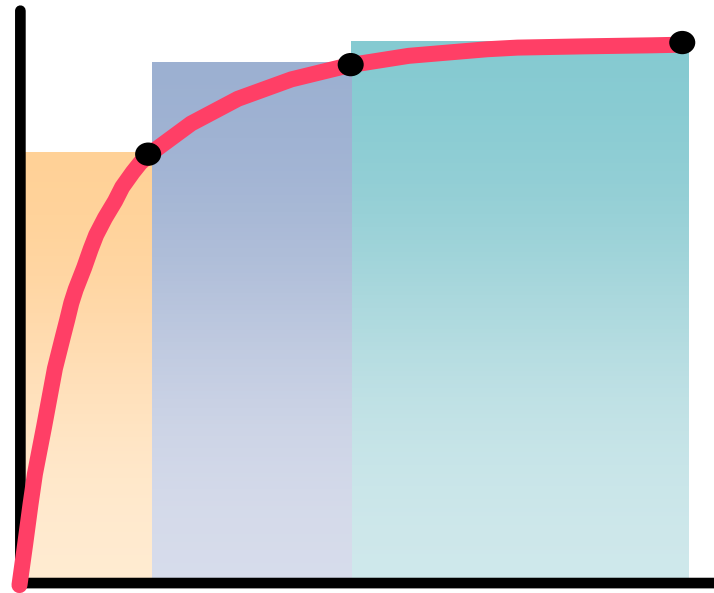
Inventory Management and more

PART IV



Tools & Decisions in Supply Chain

Inventory Management



Inventory management why ?

- When Jeff Bezos opened his revolutionary business in 1995, **Amazon.com** was intended to be a “virtual” retailer—no inventory, **no warehouses, no overhead**—just a bunch of computers taking orders for books and authorizing others to fill them.
- Now, **Amazon stocks millions of items of inventory**, amid hundreds of thousands of bins on shelves in over 150 warehouses around the world.



Why inventory ?

1. To provide a selection of goods **for anticipated customer demand** and to separate the firm from fluctuations in that demand. Such inventories are typical in retail establishments.

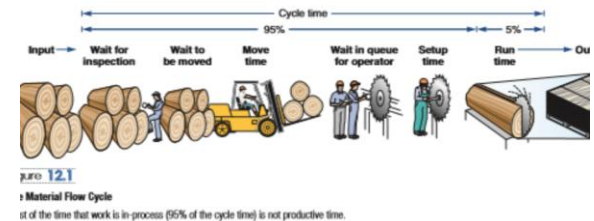
2. To **decouple various parts of the production process**. For example, if a firm's supplies fluctuate, extra inventory may be necessary to decouple the production process from suppliers.

3. To take **advantage of quantity discounts**, because purchases in larger quantities may reduce the cost of goods or their delivery.

4. To hedge **against inflation and upward price changes**

Types of inventory

- **Raw material inventory** Materials that are usually purchased but have yet to enter the manufacturing process.
- **Work-in-process (WIP) inventory** Products or components that are no longer raw materials but have yet to become finished products.
- **Maintenance/repair/operating (MRO) inventory** Maintenance, repair, and operating materials.



Inventory Costs

- Interest or Opportunity Cost
- Storage and Handling Costs
- Taxes, Insurance, and Shrinkage



Inventory Costs

- Customer Service
- Ordering Cost
- Setup Cost
- Labor and Equipment Utilization
- Transportation Costs
- Payments to Suppliers



Types of Inventory

Cycle Inventory

$$\text{Average cycle inventory} = \frac{Q + 0}{2}$$

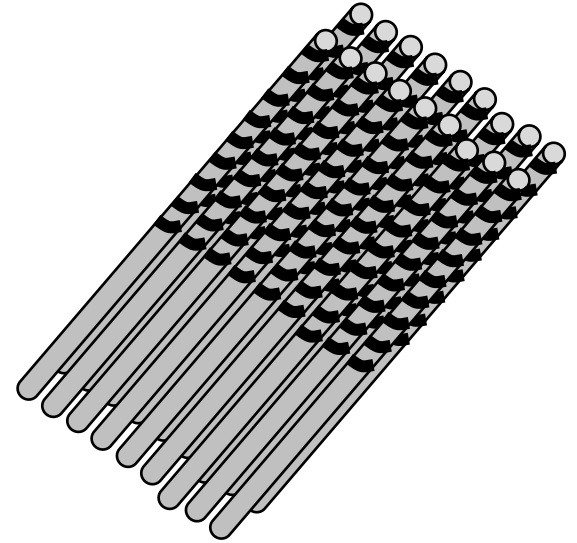
Safety Stock Inventory

Anticipation Inventory

Pipeline Inventory

$$\text{Pipeline inventory} = D_L^{\bar{}} = dL$$

Types of Inventory

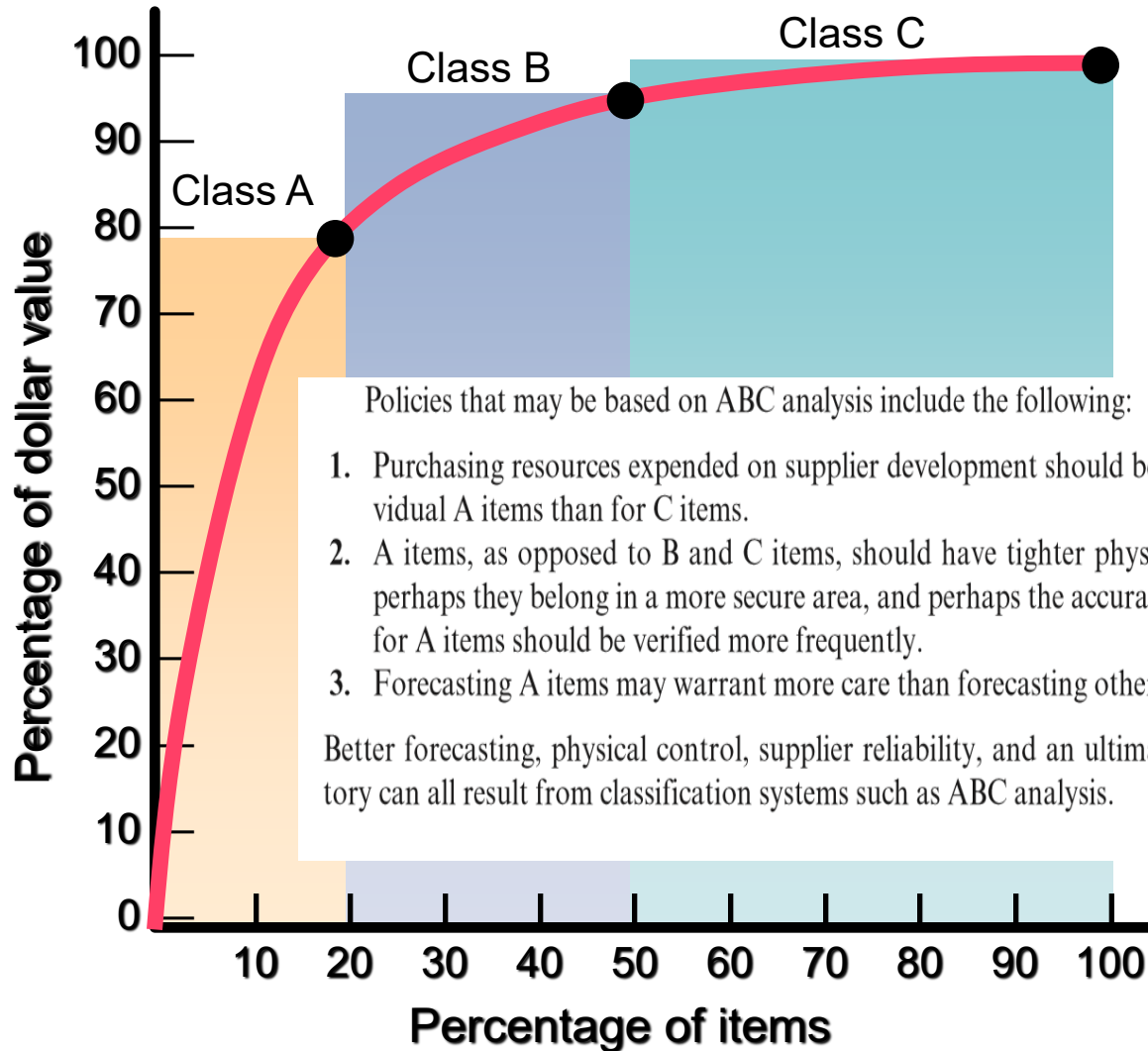


$$\begin{aligned}\text{Cycle inventory} &= Q/2 \\ &= 280/2 \\ &= 140 \text{ drills}\end{aligned}$$

$$\begin{aligned}\text{Pipeline inventory} &= \bar{D}_L = dL \\ &= (70 \text{ drills/week})(3 \text{ weeks}) \\ &= 210 \text{ drills}\end{aligned}$$

Example 15.1

ABC Analysis



How
Much?
When!



Record accuracy

And Cycle counting

- A continuing reconciliation of inventory with inventory records

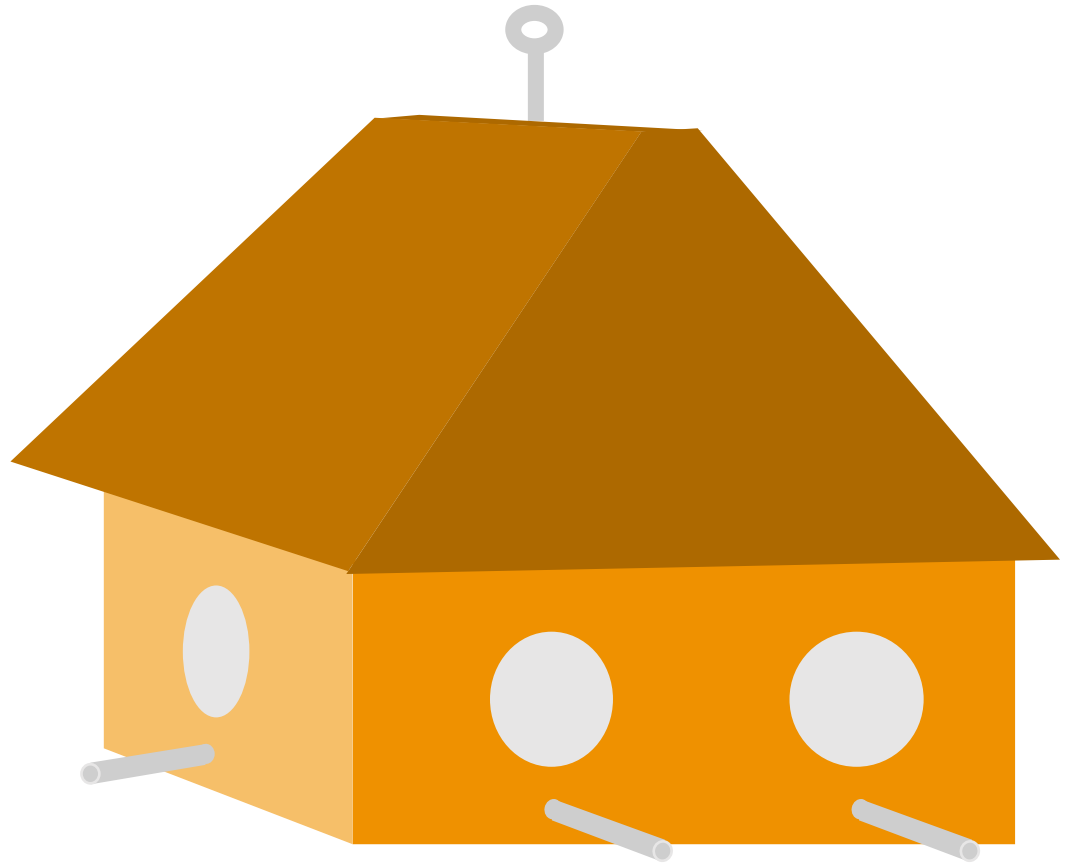


In this hospital, these vertically rotating storage carousels provide rapid access to hundreds of critical items and at the same time save floor space. This Omnicell inventory management carousel is also secure and has the added advantage of printing bar code labels.

Cycle counting example (pipeline)

- CYCLE COUNTING AT COLE'S TRUCKS, INC. Cole's Trucks, Inc., a builder of high-quality refuse trucks, has **about 5,000 items in its inventory**. It wants to determine how many items to cycle count each day.
- APPROACH After hiring Matt Clark, a bright young OM student, for the summer, the firm determined that it has
 - **500 A items, 1,750 B items, and 2,750 C items.**
- Company policy is to count **all A items every month (every 20 working days)**, **all B items every quarter (every 60 working days)**, and **all C items every 6 months (every 120 working days)**. The firm then allocates some items to be counted each day.

*Economic
Order
Quantity*



Economic Order Quantity



Assumptions

- 1. Demand rate is constant**
- 2. No constraints on lot size**
- 3. Only relevant costs are holding and ordering/setup**
- 4. Decisions for items are independent from other items**
- 5. No uncertainty in lead time or supply**

Economic Order Quantity

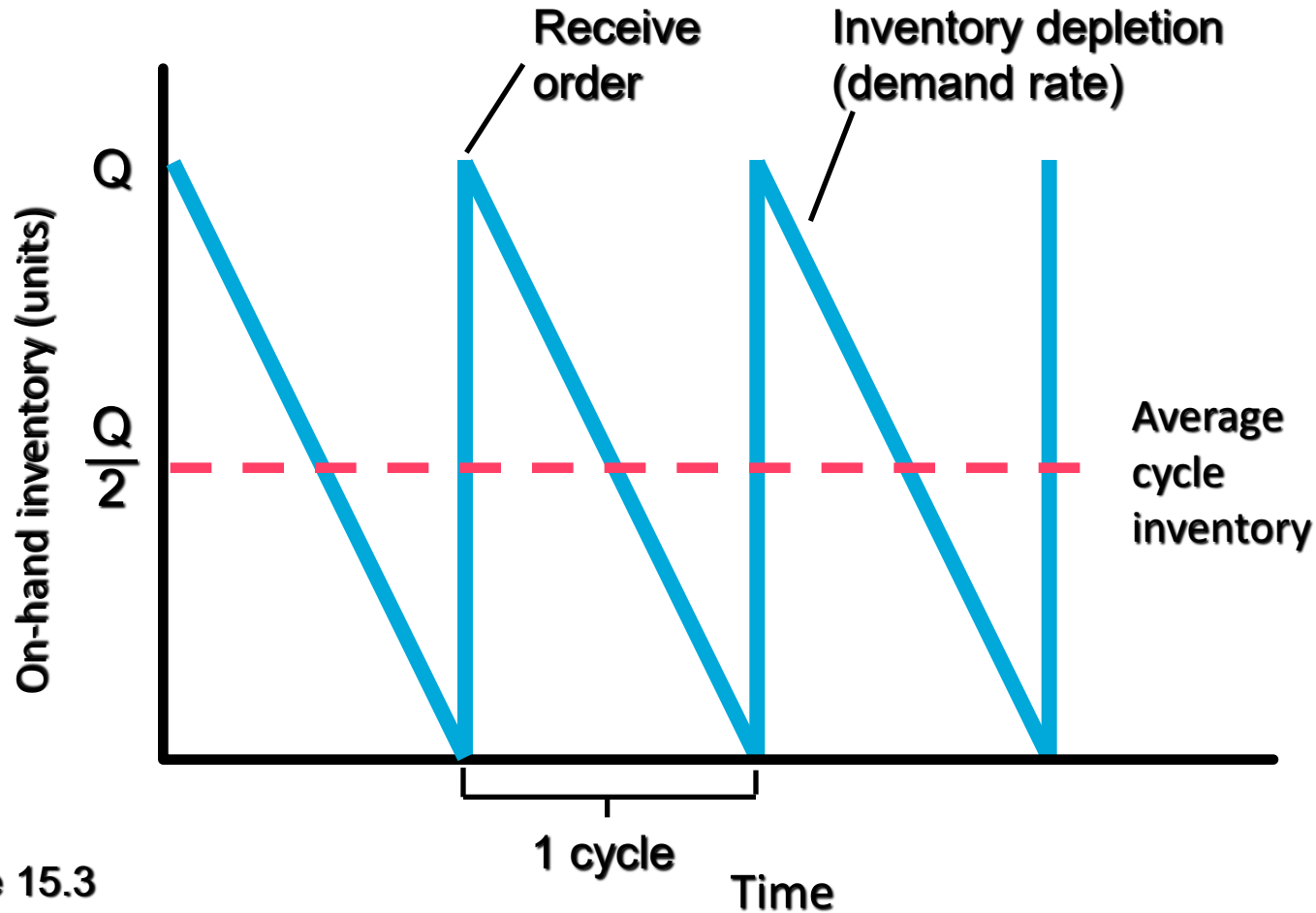


Figure 15.3

Economic Order Quantity

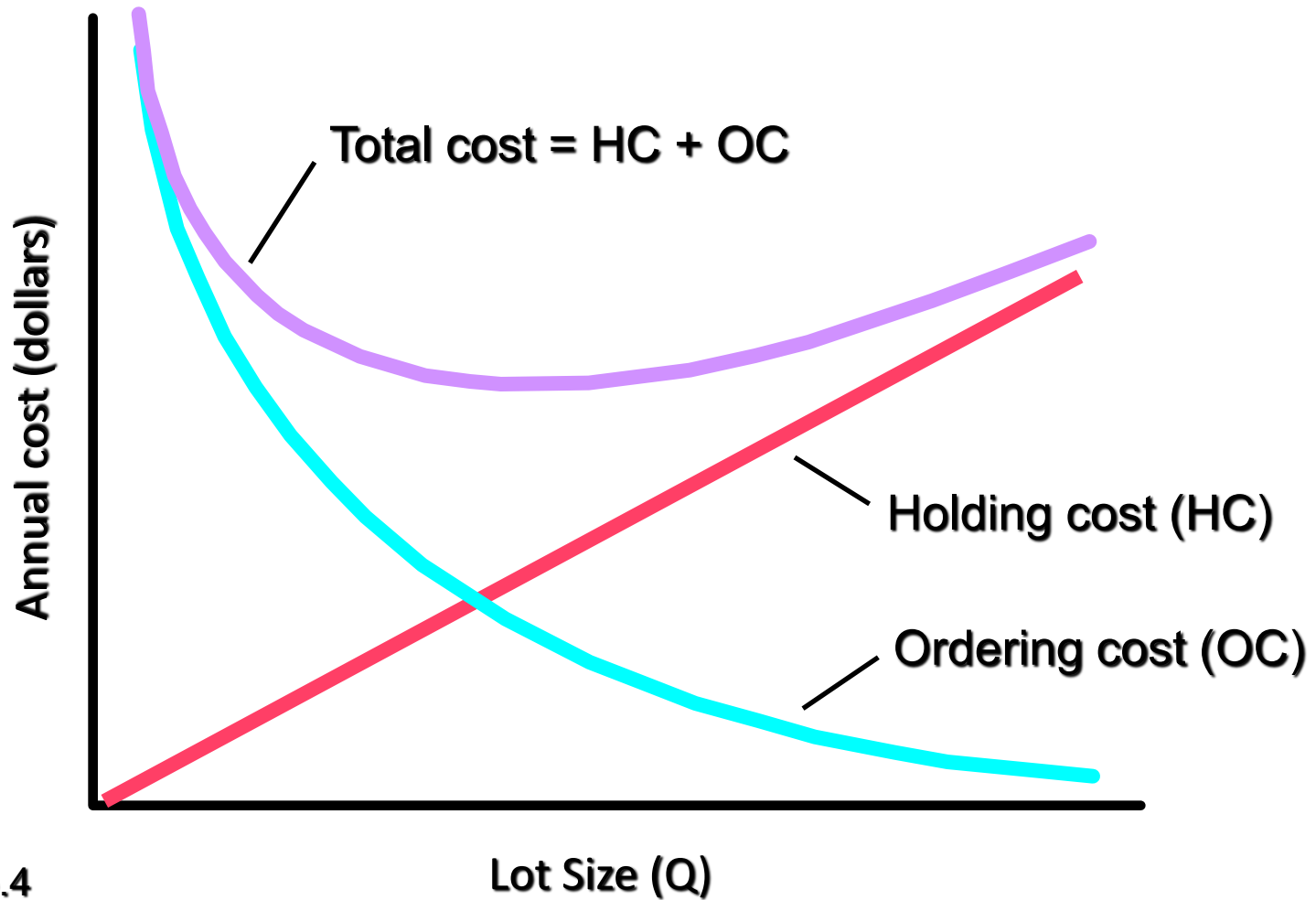


Figure 15.4

EXAMPLE

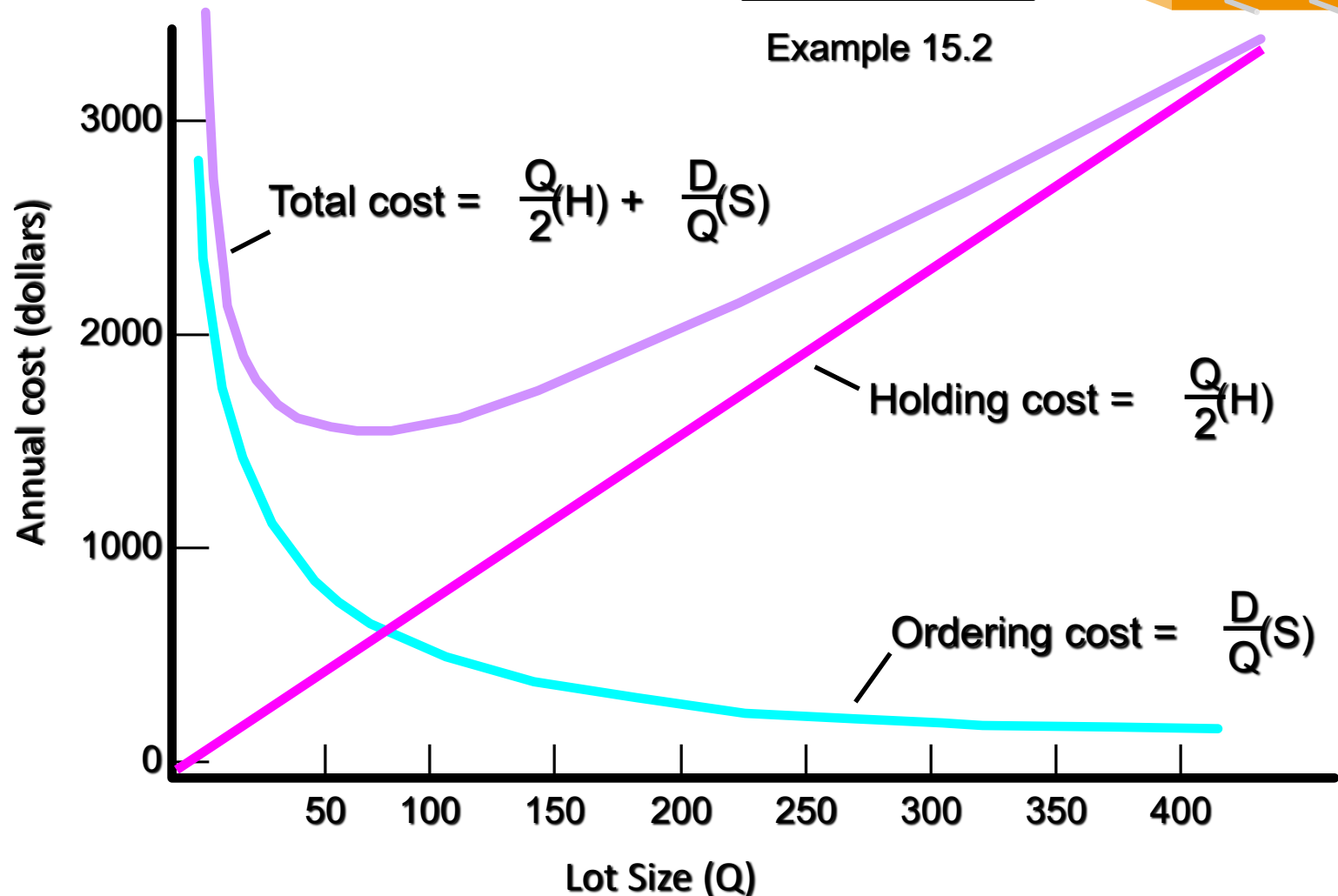
Total cost is Holding cost + ordering cost

- Costing **Out a Lot-Sizing Policy**
- A museum of natural history opened a gift shop two years ago. Managing inventories has become a problem. Low inventory turnover is squeezing profit margins and causing cash-flow *problems*.
- One of the top-selling items in the container group at the museum's gift shop is a bird feeder. **Sales are 18 units per week, and the supplier charges \$60 per unit. The cost of placing an order with the supplier is \$45. Annual holding cost is 25 percent of a feeder's value, and the museum operates 52 weeks per year.** Management chose a 390-unit lot size so that new orders could be placed less frequently. What is the annual cost of the current policy of using a 390-unit lot size? Would a lot size of 468 be better?
-

Economic Order Quantity



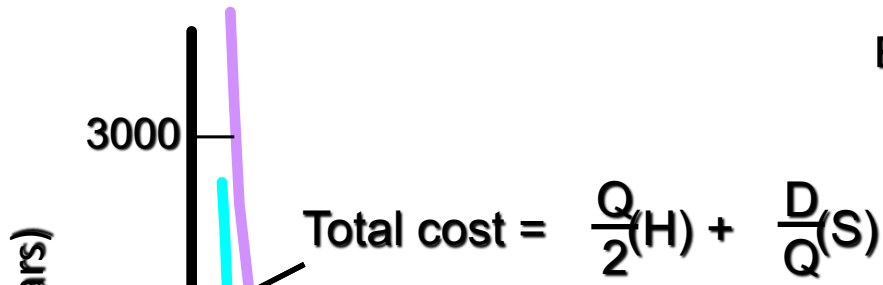
Example 15.2



Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

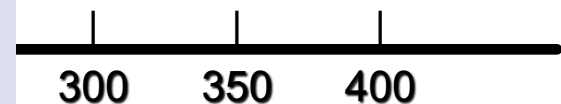
$$S = \$45 \qquad Q = 390 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

$$\text{Holding cost} = \frac{Q}{2}(H)$$

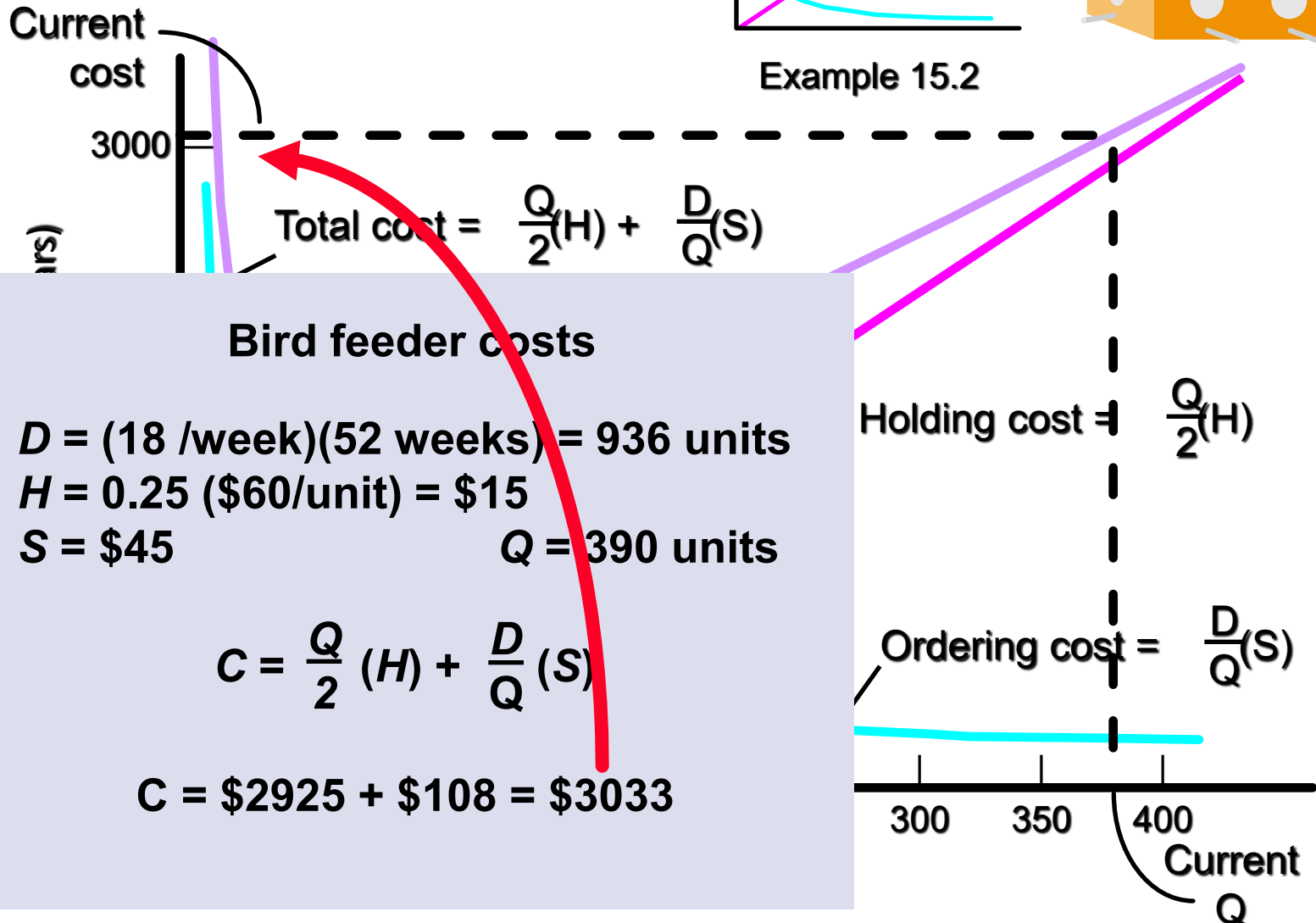
$$\text{Ordering cost} = \frac{D}{Q}(S)$$



Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

$$S = \$45 \quad Q = 390 \text{ units}$$

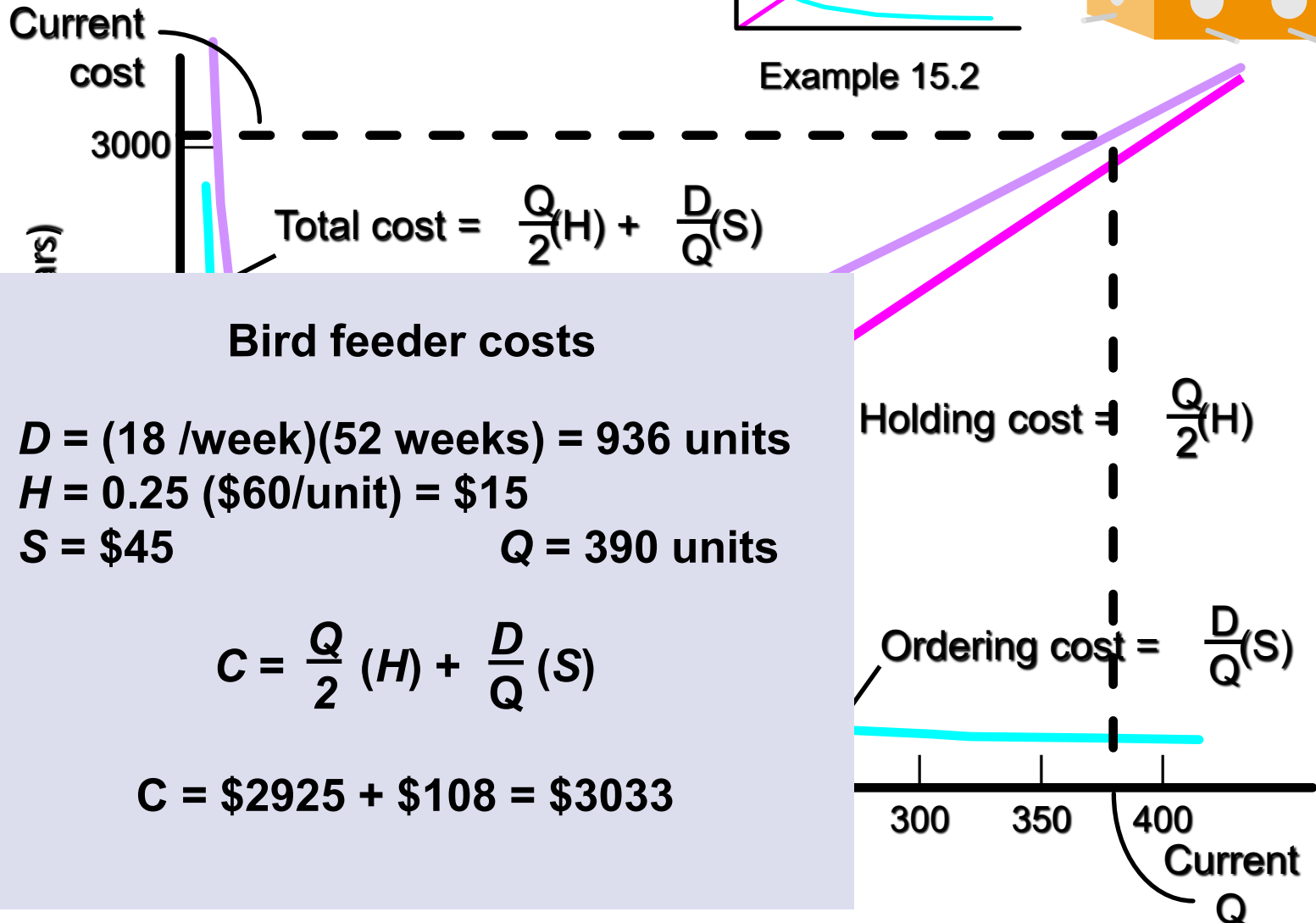
$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

$$S = \$45 \qquad Q = 390 \text{ units}$$

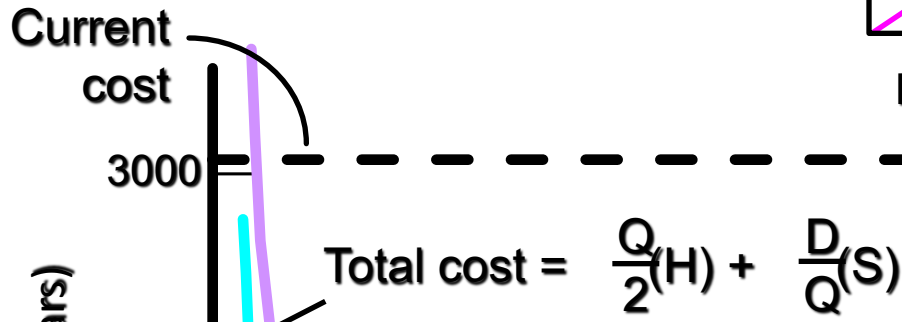
$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \$2925 + \$108 = \$3033$$

Economic Order Quantity



Example 15.2



Bird feeder costs

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

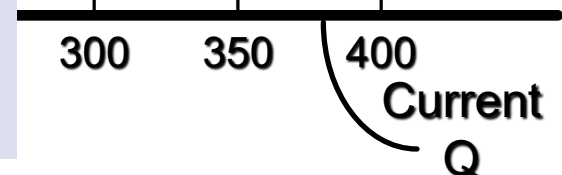
$$S = \$45 \quad Q = 468 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

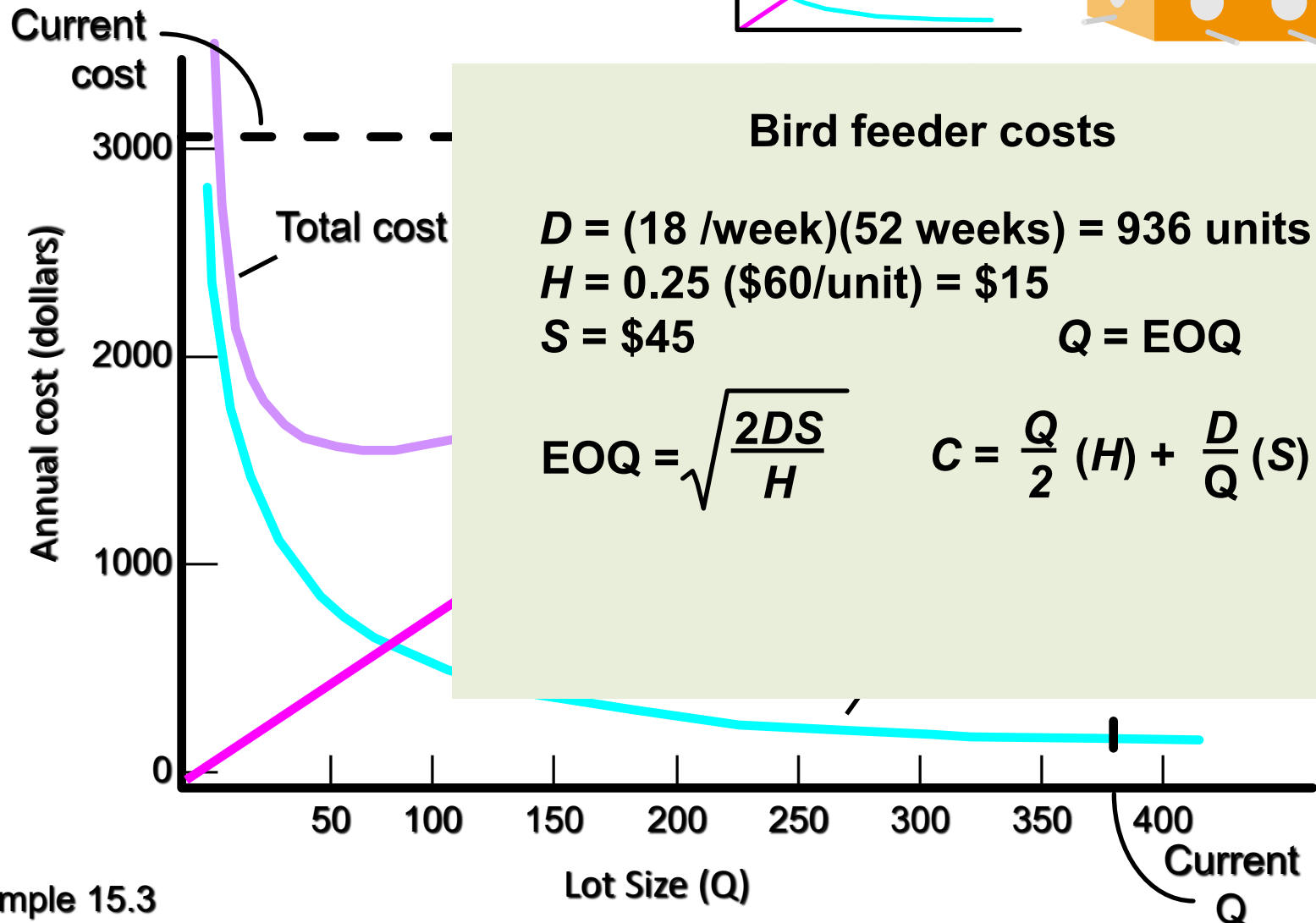
$$C = \$3510 + \$90 = \$3600$$

$$\text{Holding cost} = \frac{Q}{2}(H)$$

$$\text{Ordering cost} = \frac{D}{Q}(S)$$

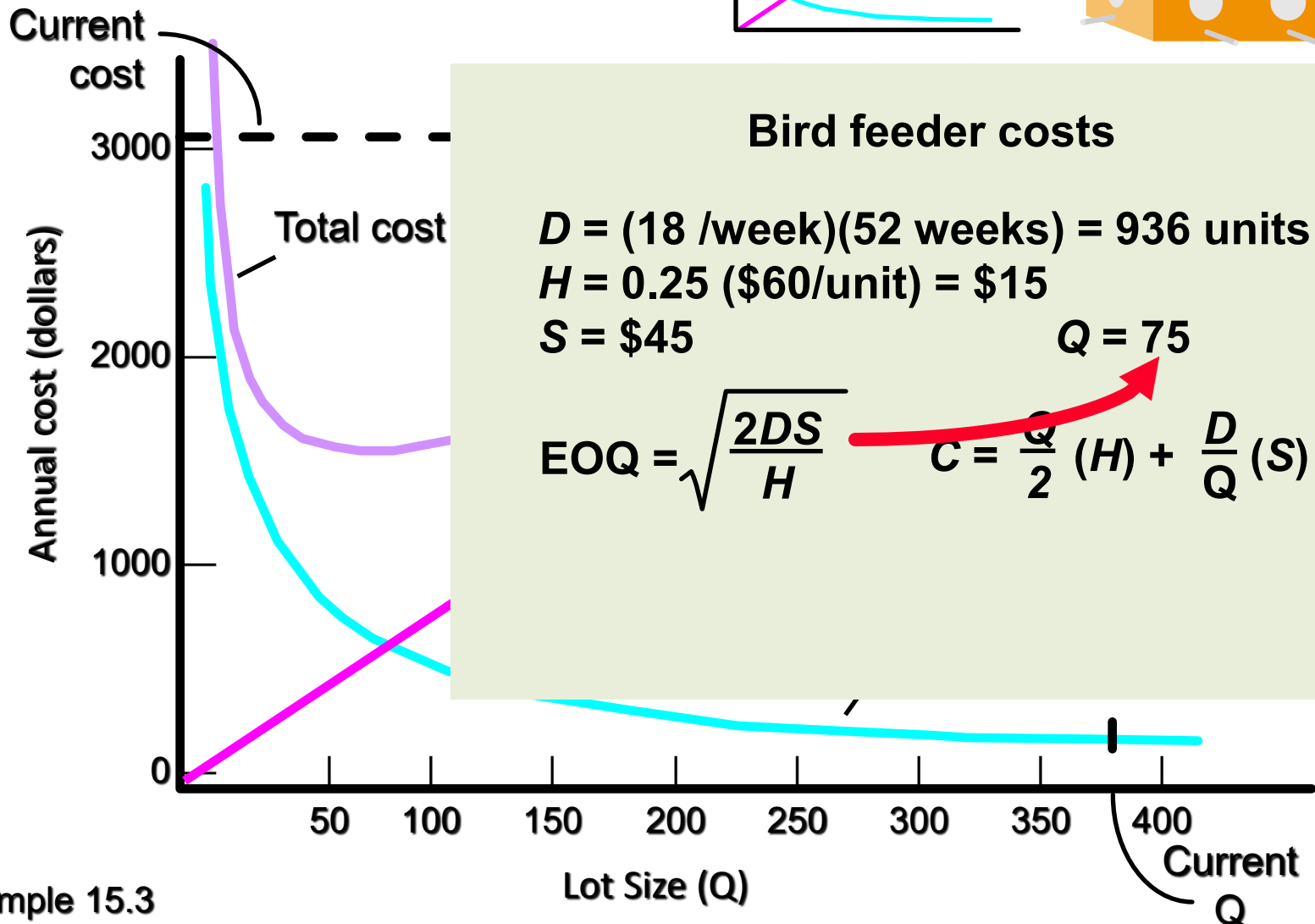


Economic Order Quantity



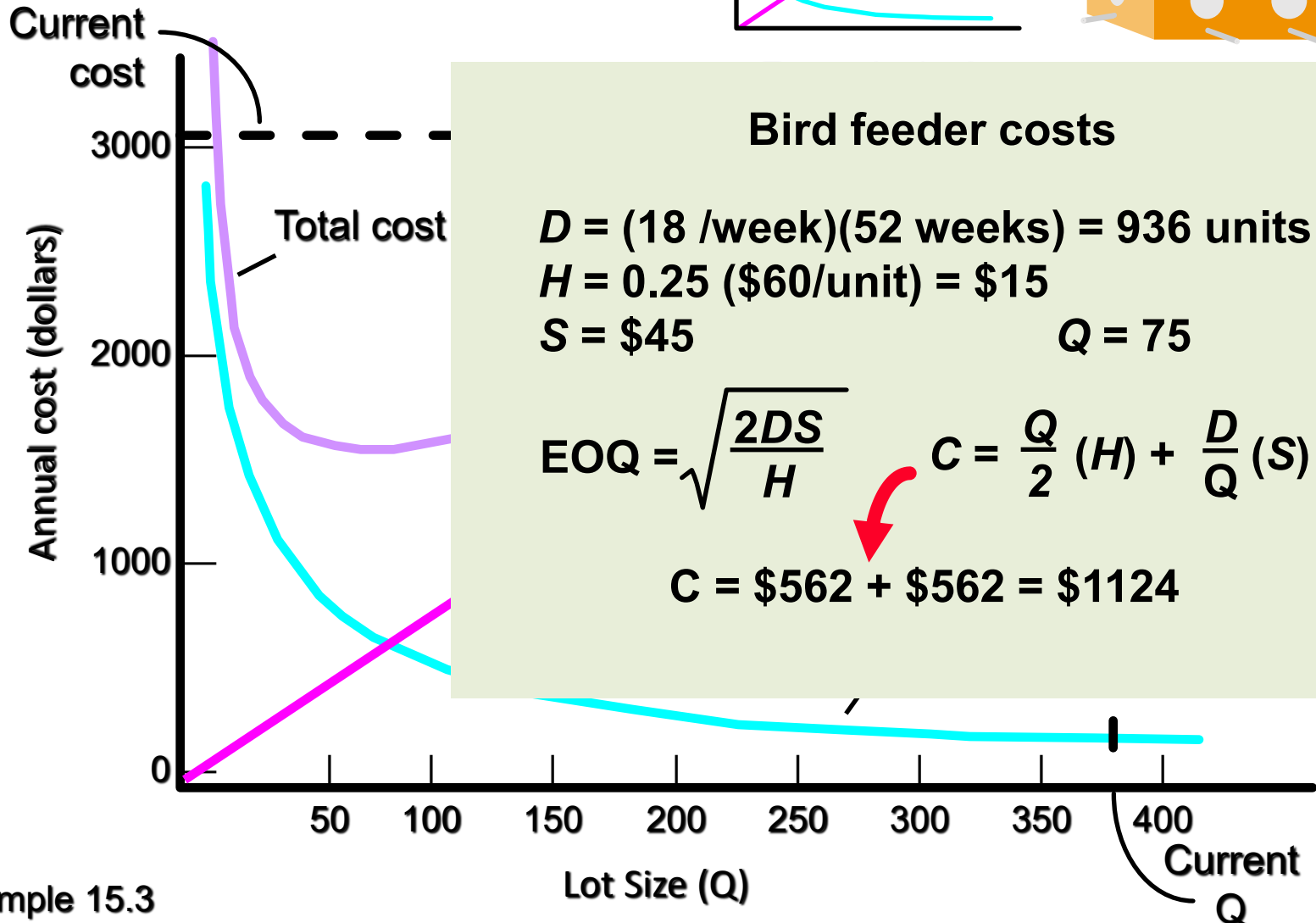
Example 15.3

Economic Order Quantity



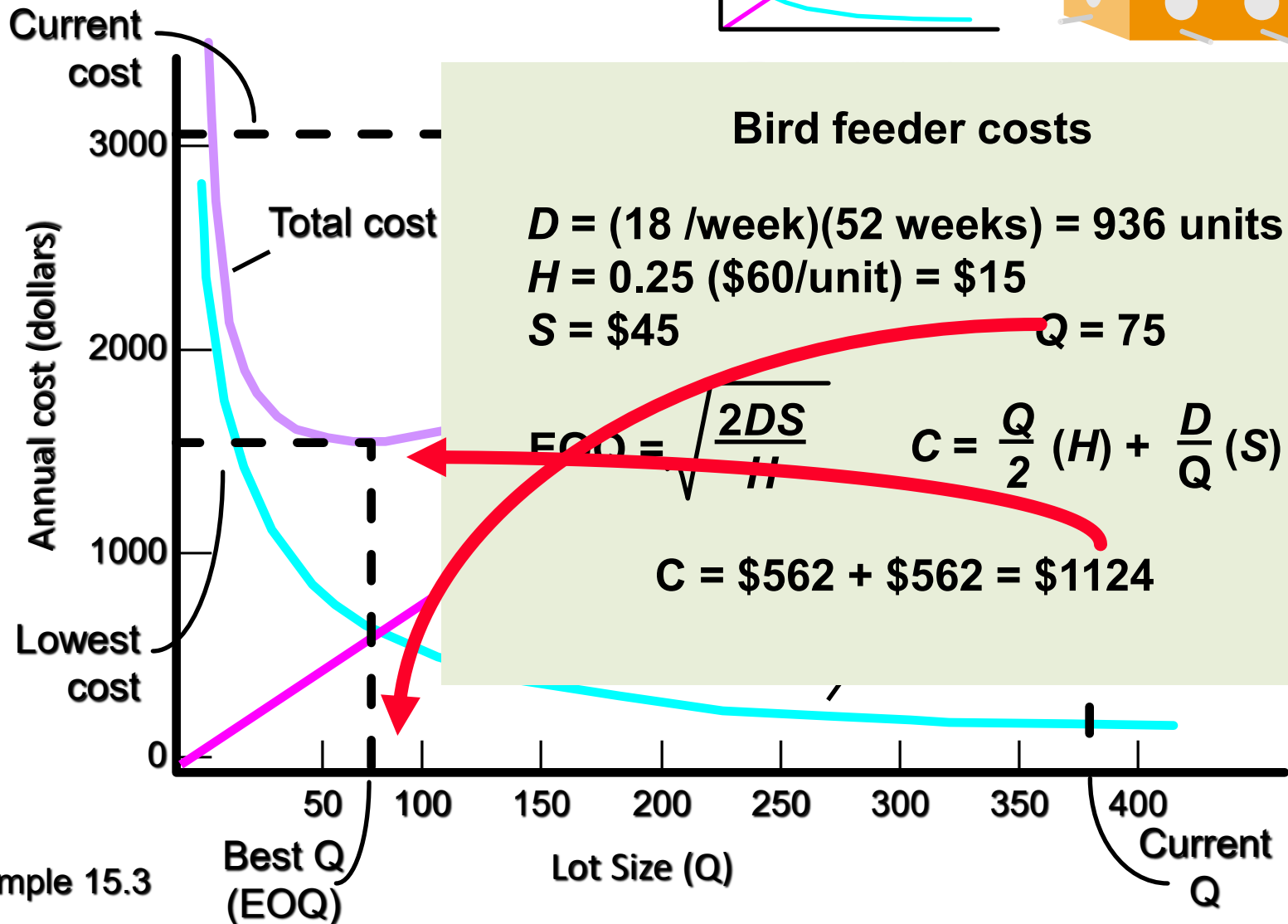
Example 15.3

Economic Order Quantity



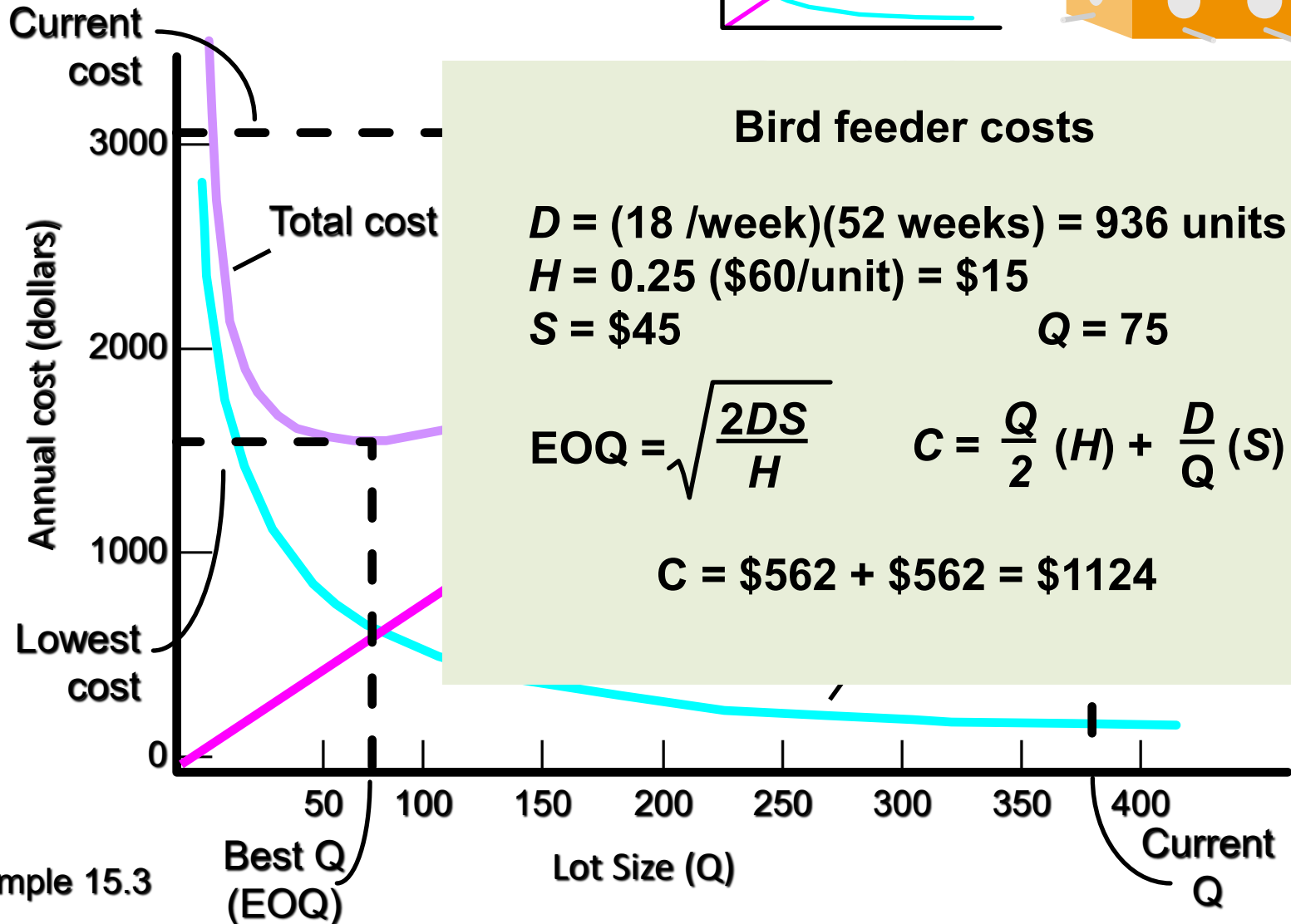
Example 15.3

Economic Order Quantity



Example 15.3

Economic Order Quantity



Example 15.3

Economic Order Quantity



Current
cost



Bird feeder costs

Parameters

Current Lot Size (Q)	390
Demand (D)	936
Order Cost (S)	\$45
Unit Holding Cost (H)	\$15

Economic Order Quantity 75

Annual Costs

Orders per Year	2.4
Annual Ordering Cost	\$108.00
Annual Holding Cost	\$2,925.00
Annual Inventory Cost	\$3,033.00

Annual Costs based on EOQ

Orders per Year	12.48
Annual Ordering Cost	\$561.60
Annual Holding Cost	\$562.50
Annual Inventory Cost	\$1,124.10

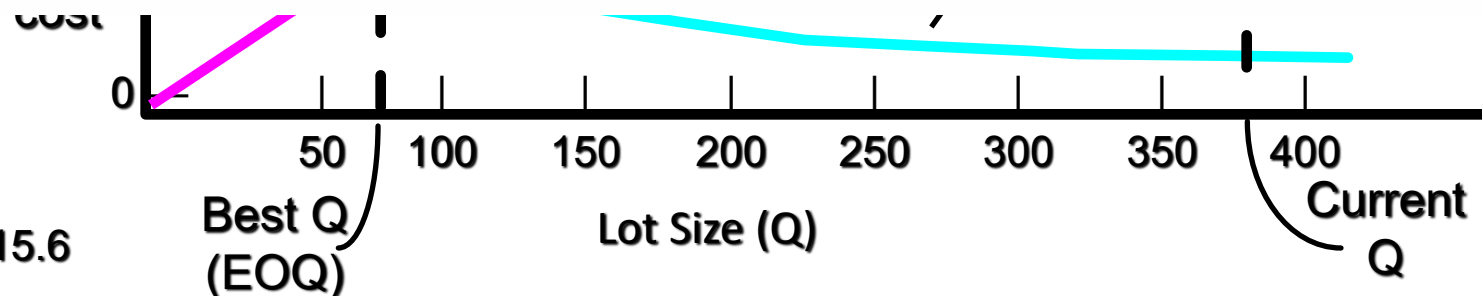
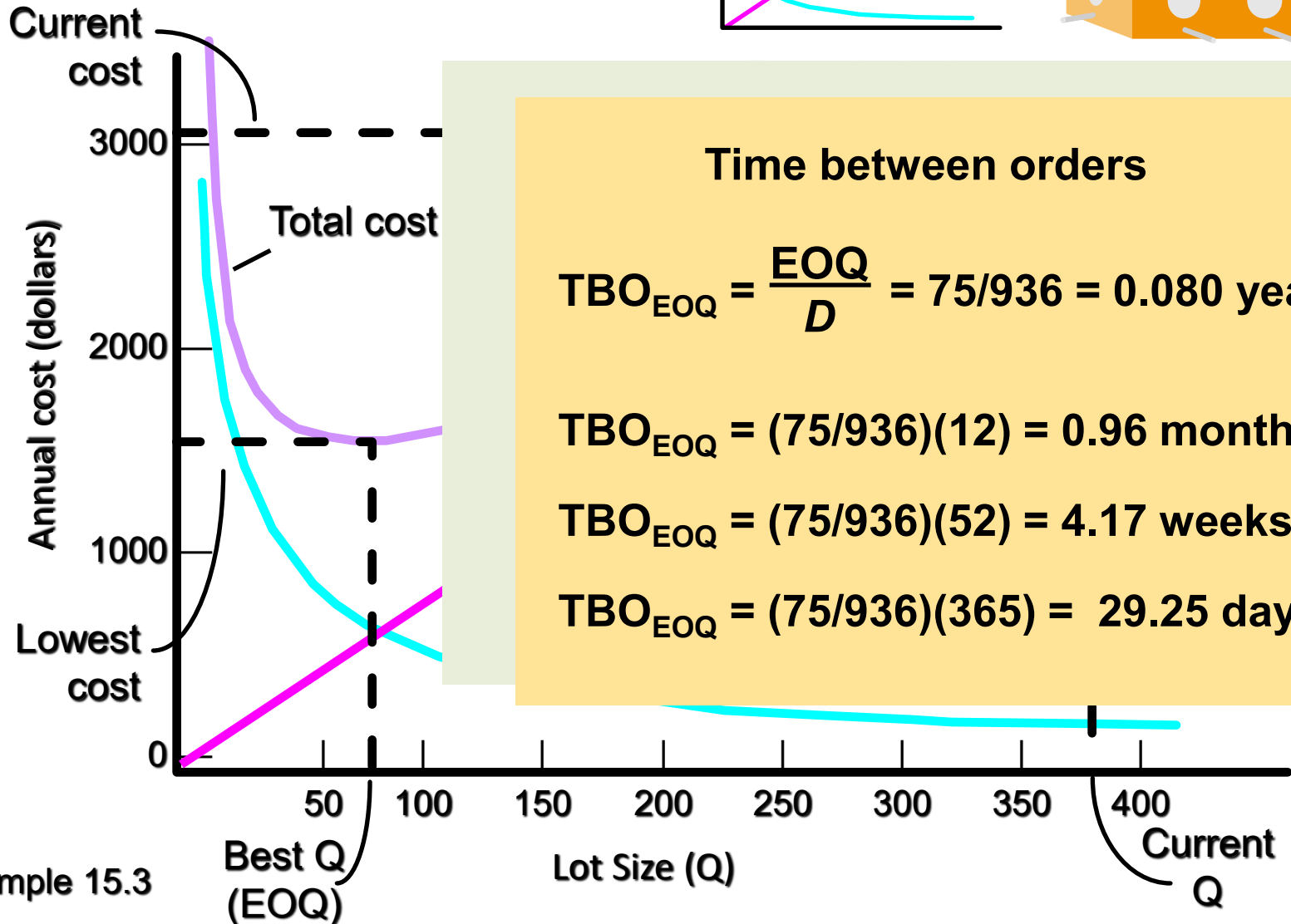


Figure 15.6

Economic Order Quantity



How
Much?
When!



Continuous Review

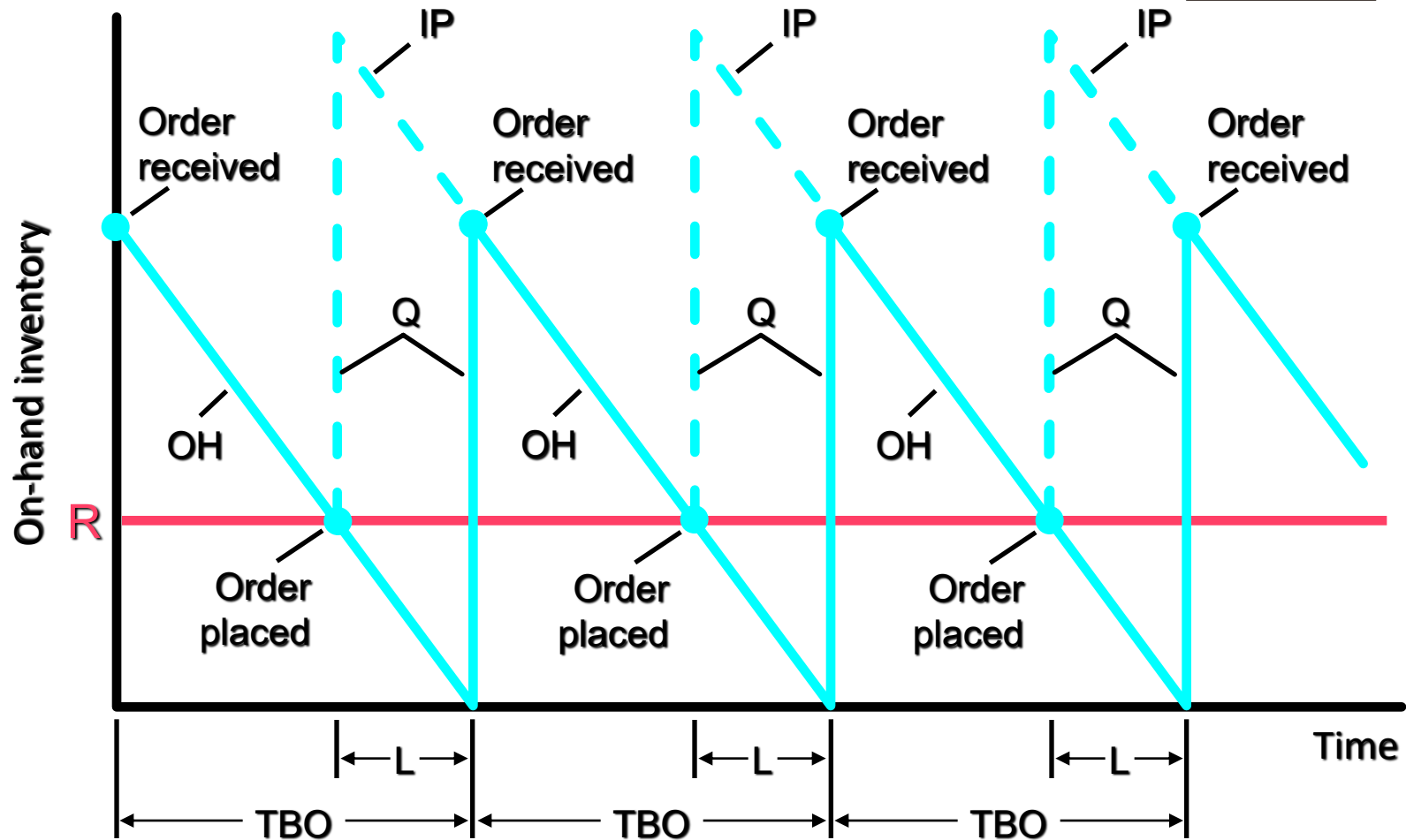
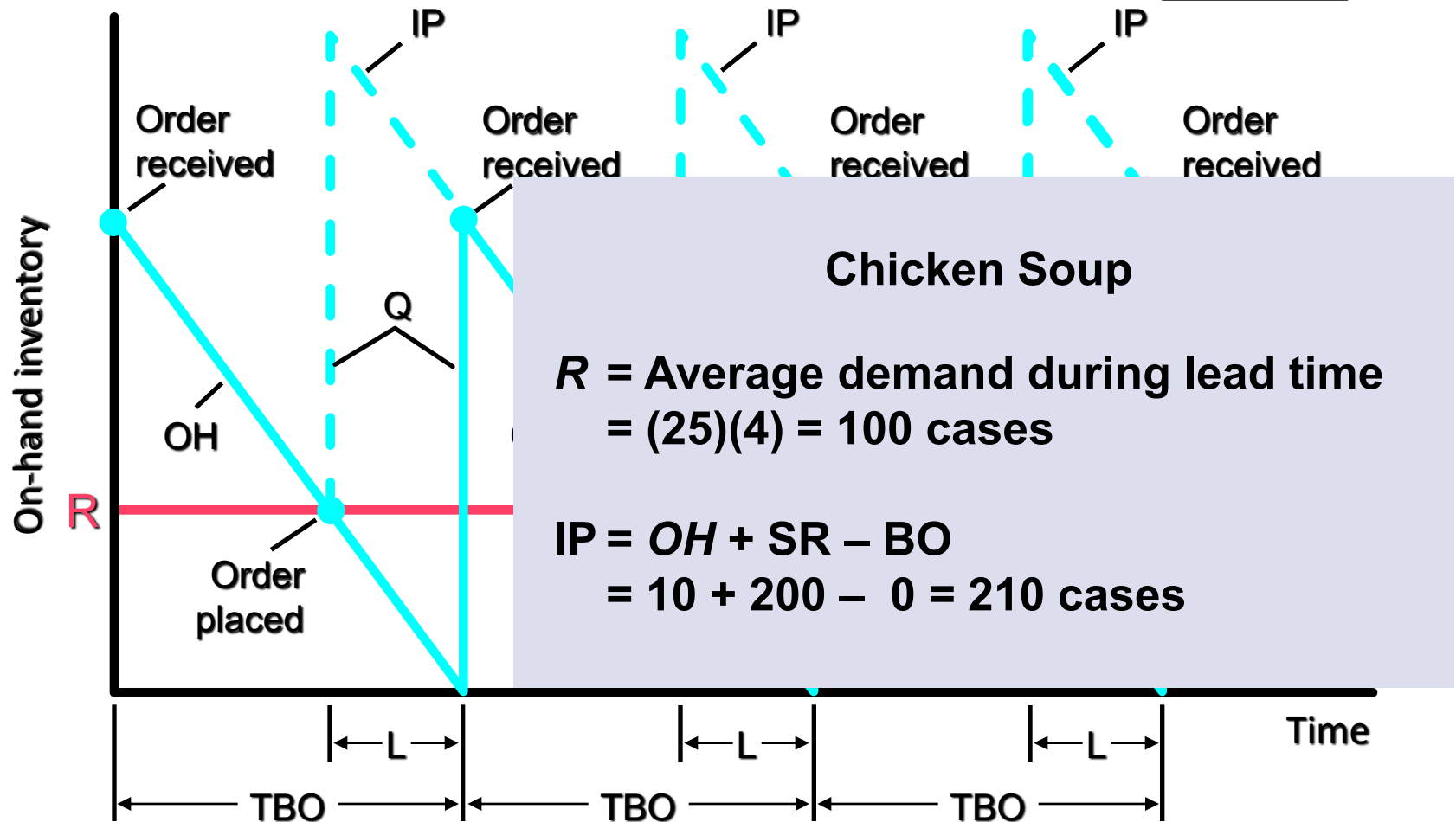


Figure 15.7

EXAMPLE

- **Determining Whether to Place an Order**
 - Demand for chicken soup at a supermarket is always **25 cases a day** and **the lead time is always four days**. The shelves were just restocked with chicken soup, leaving an on-hand inventory of only 10 cases. There are no backorders, but there is one open order for 200 cases. What is the inventory position? Should a new order be placed?

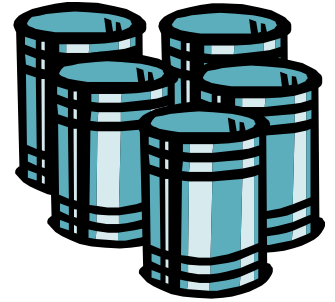
Continuous Review



Example 15.4

Special Inventory Models

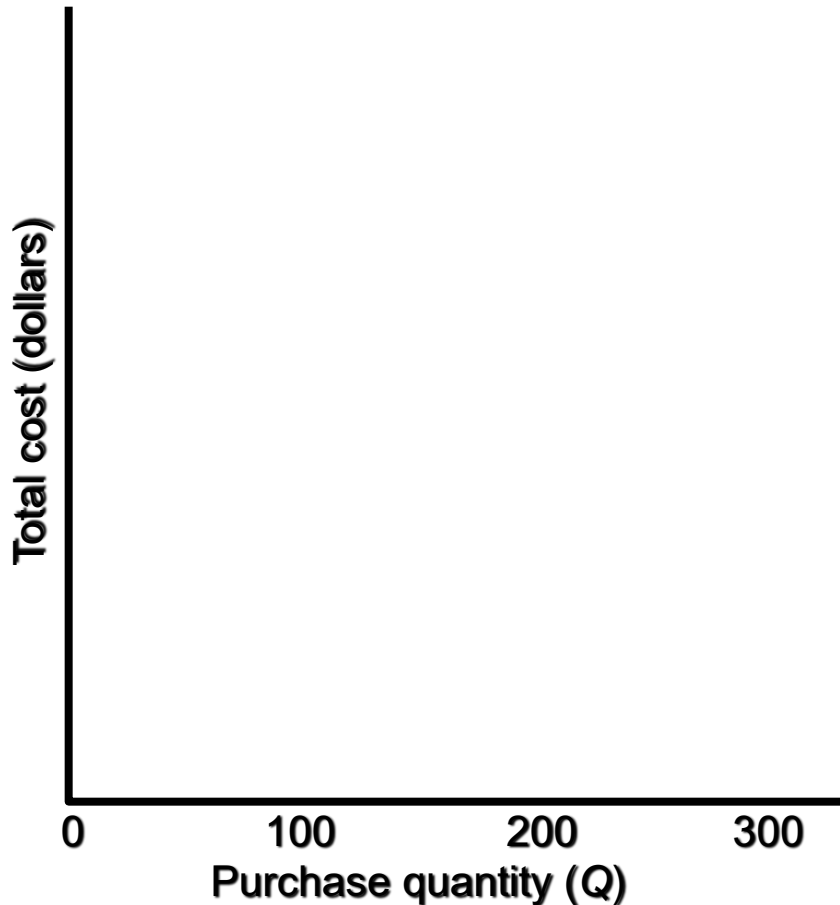
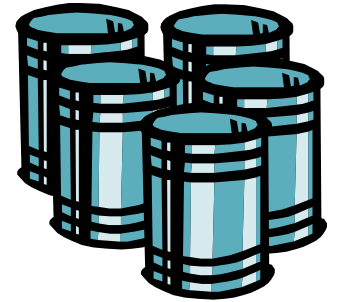
Quantity Discounts



Special Inventory Models

Quantity Discounts

Figure E.3

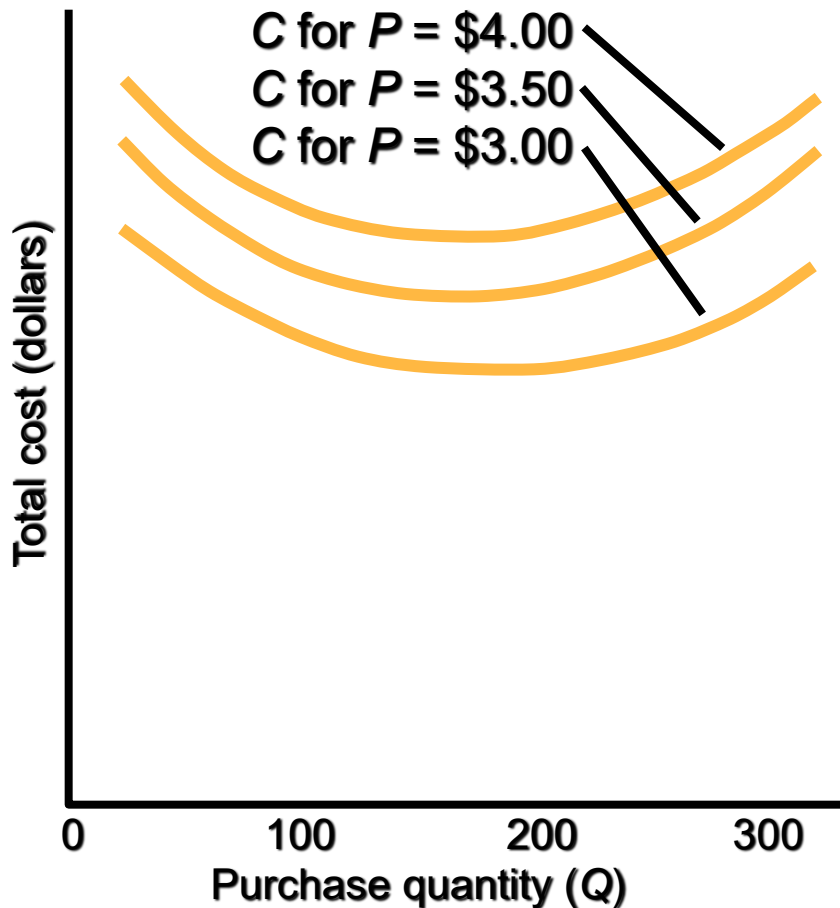
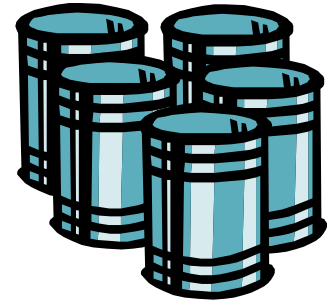


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

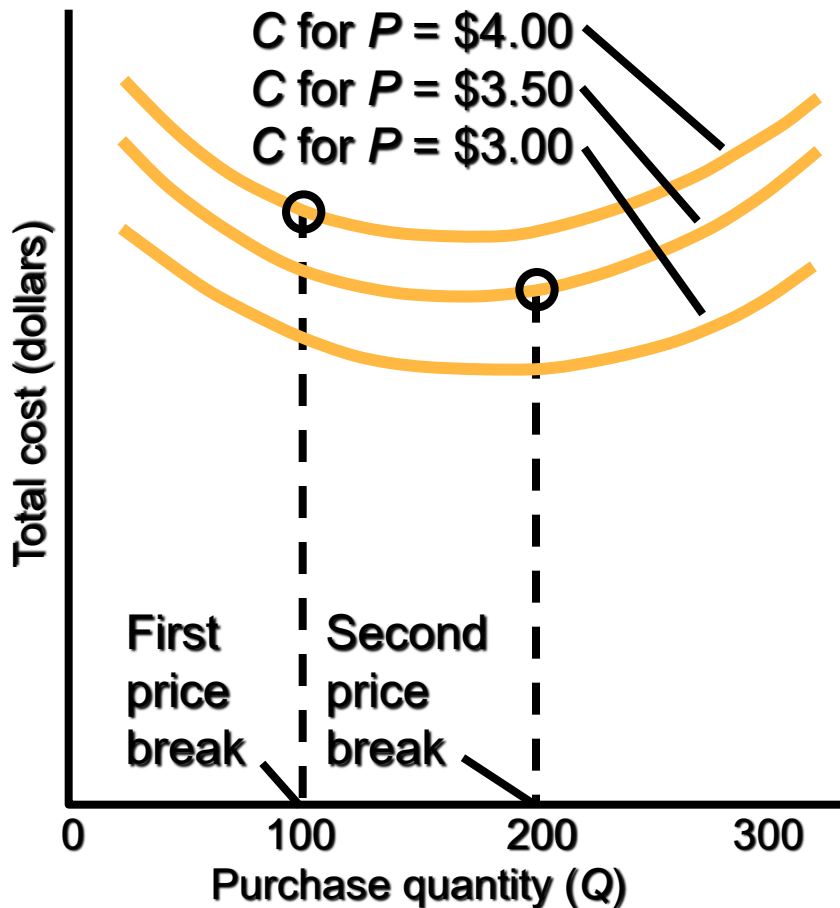
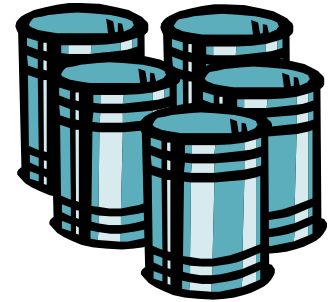


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

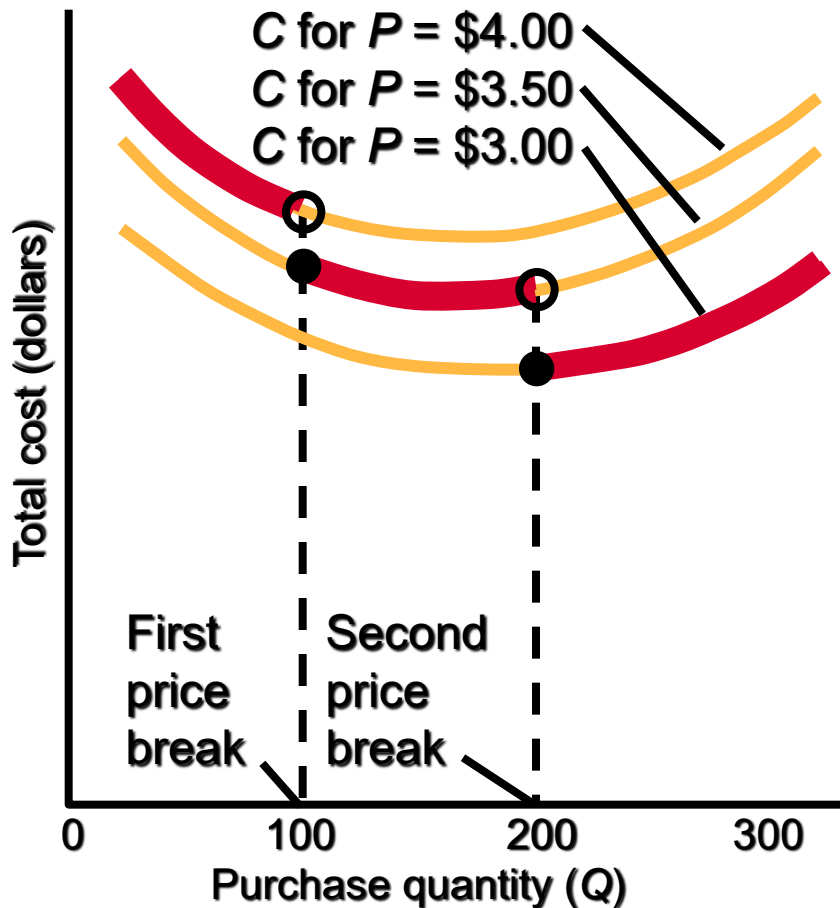
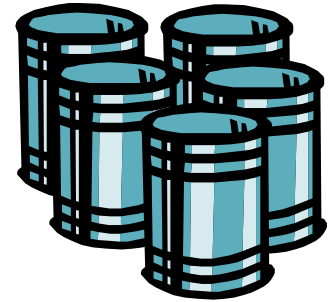


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

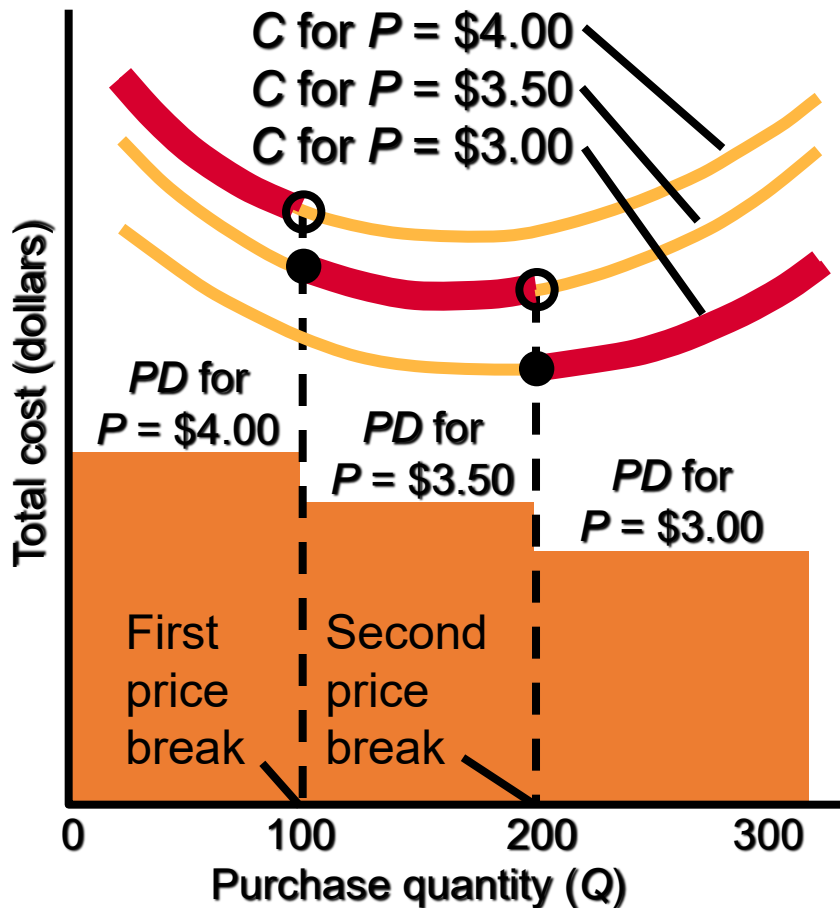
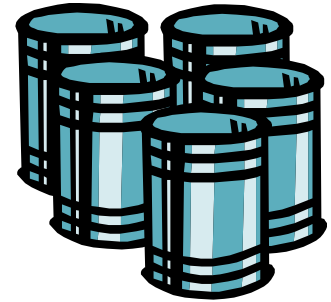


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

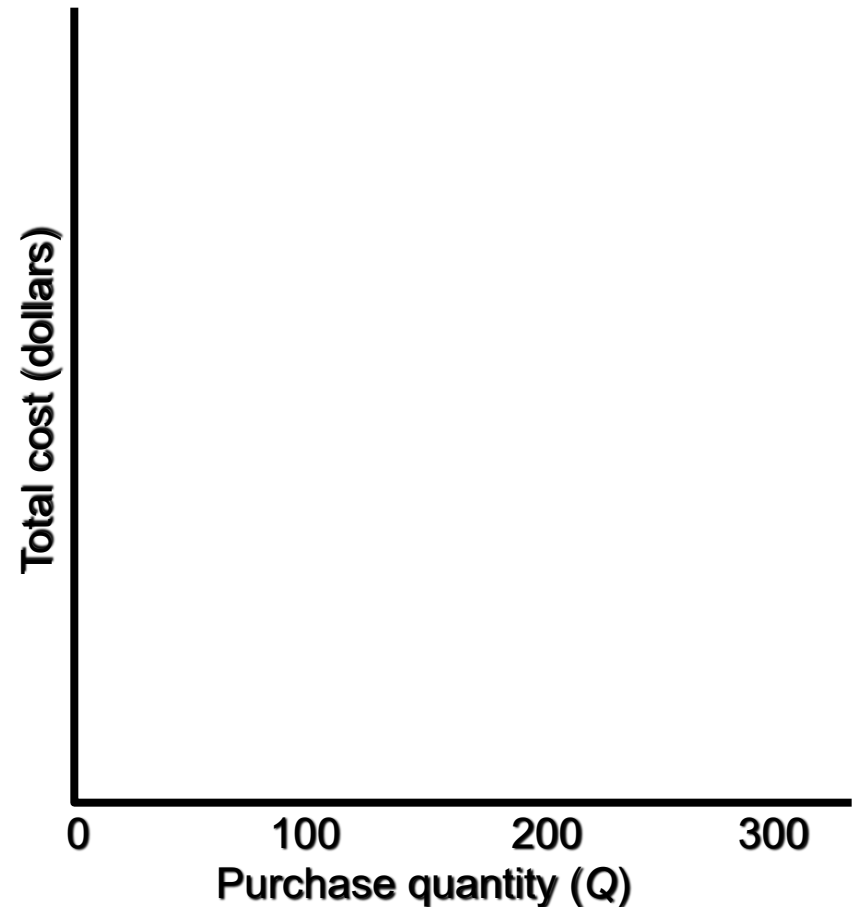
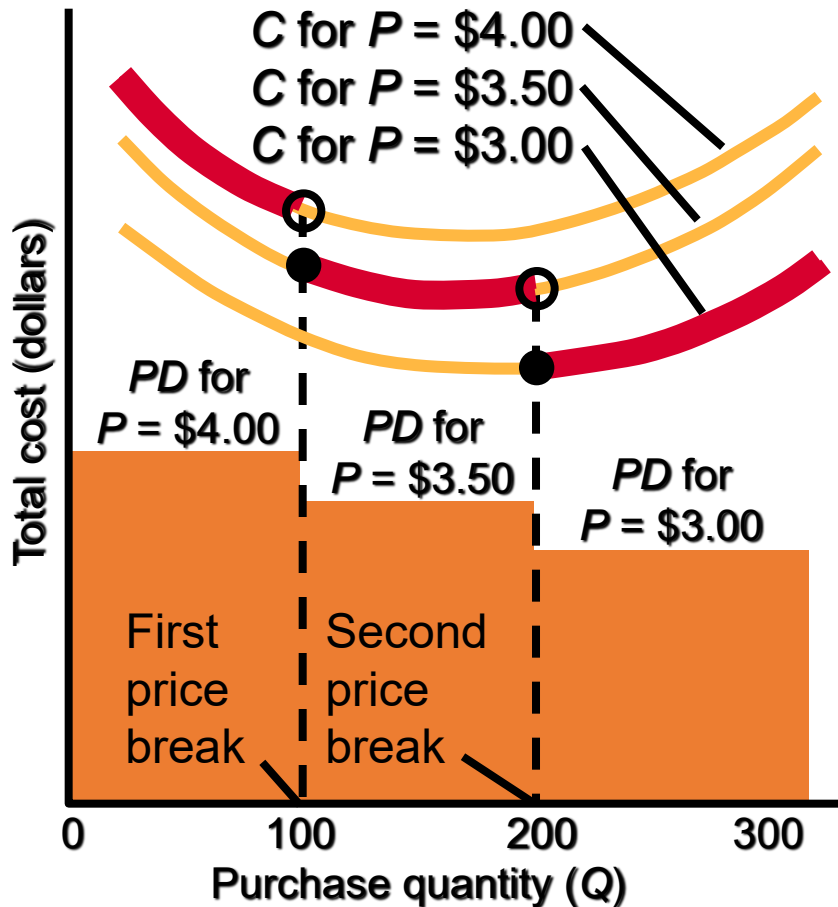
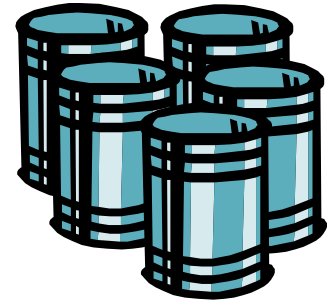


(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3

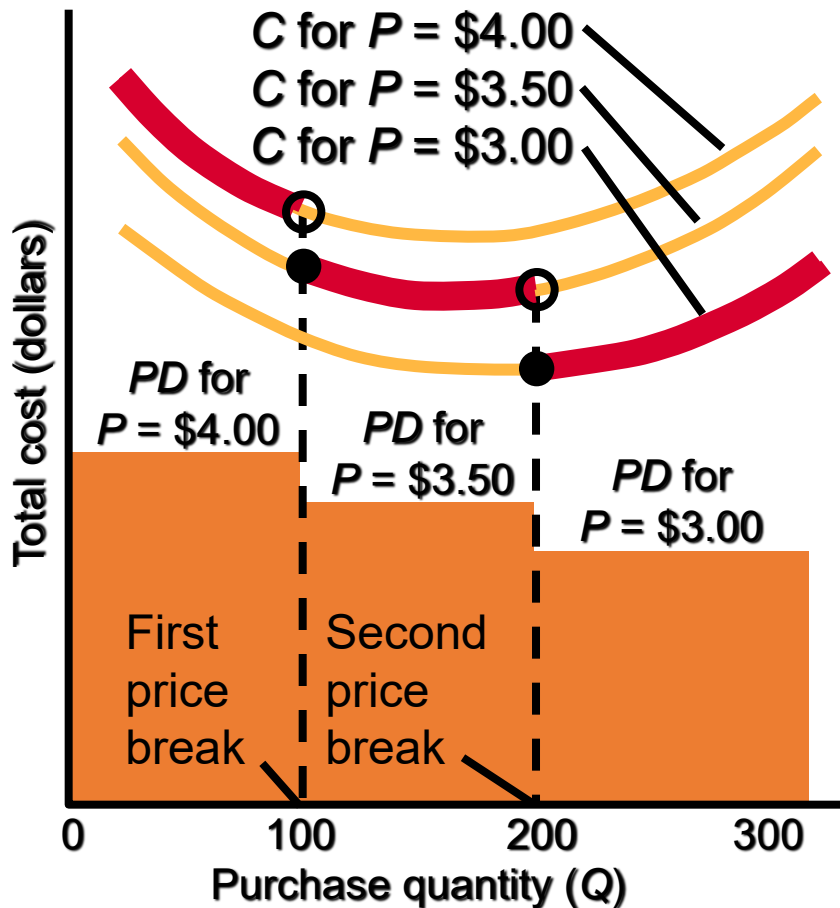
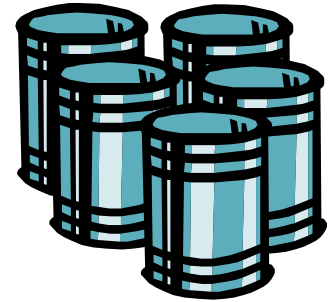


(b) EOQs and price break quantities

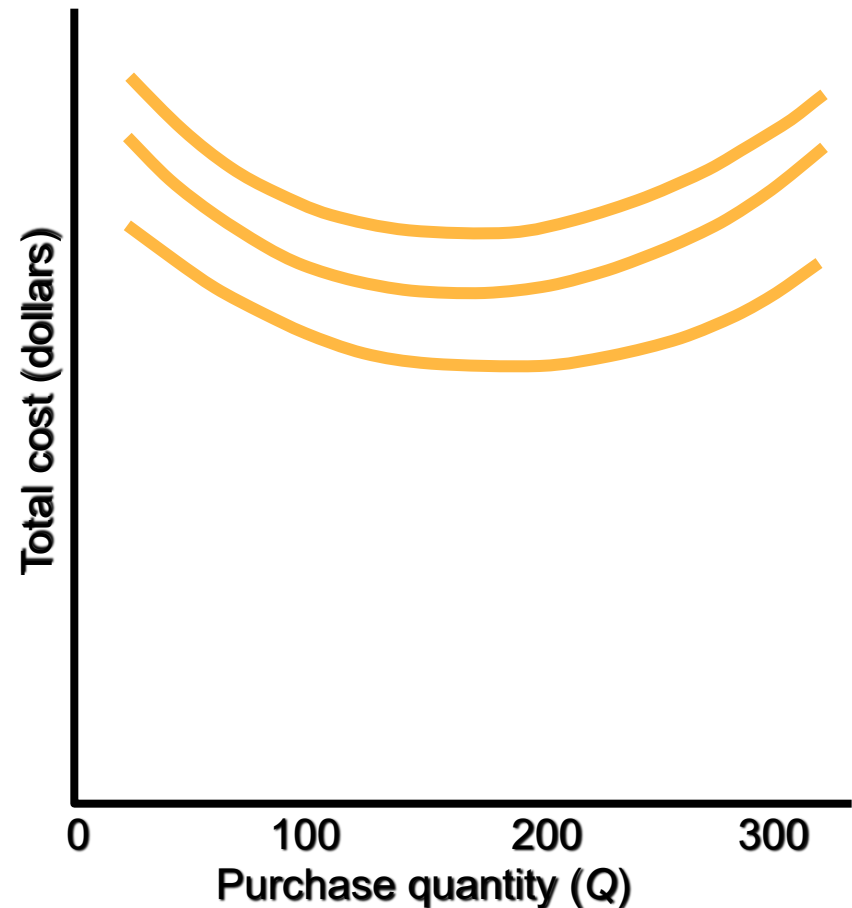
Special Inventory Models

Quantity Discounts

Figure E.3



(a) Total cost curves with purchased materials added

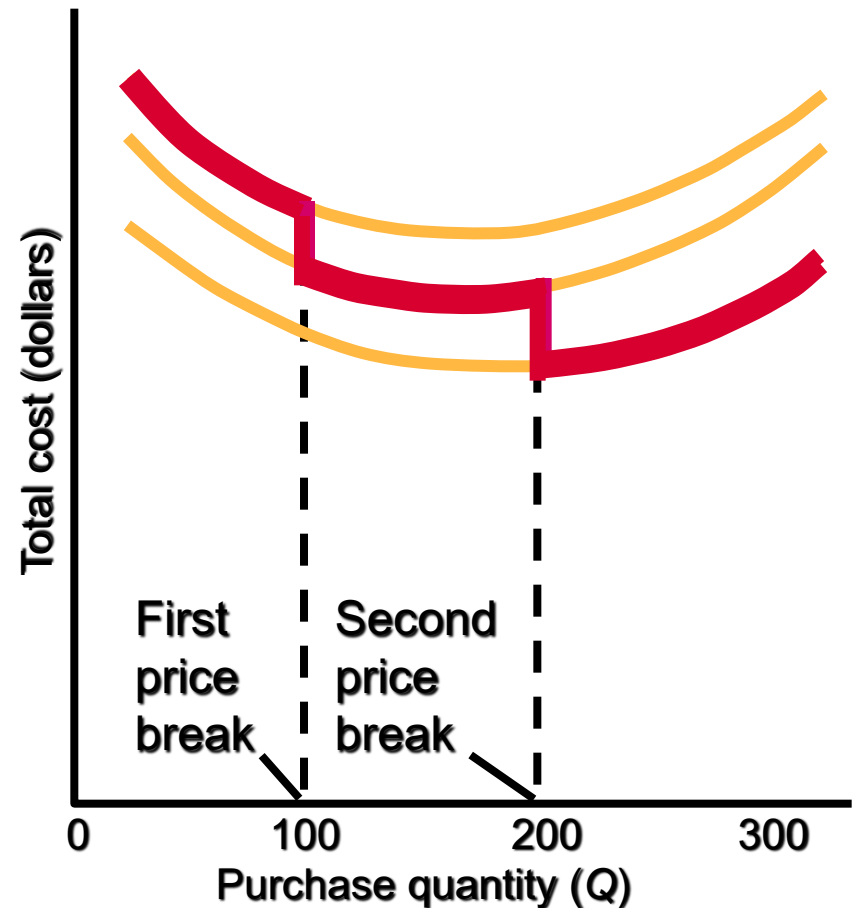
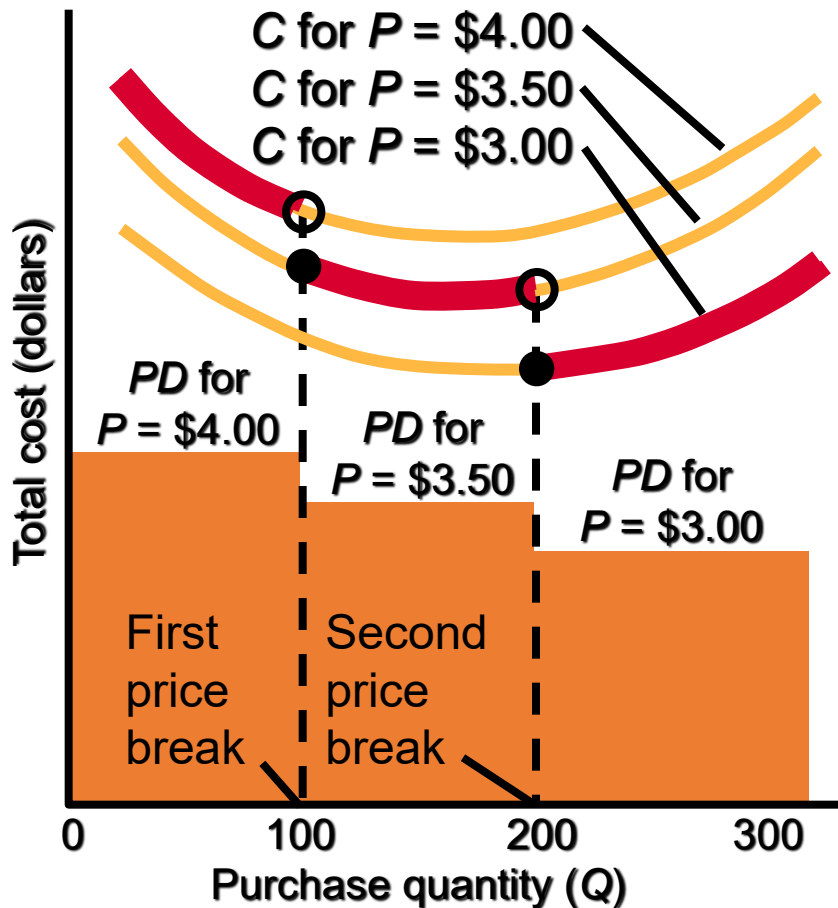
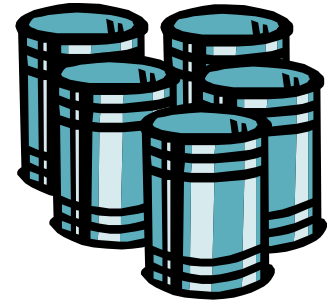


(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

Figure E.3



(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

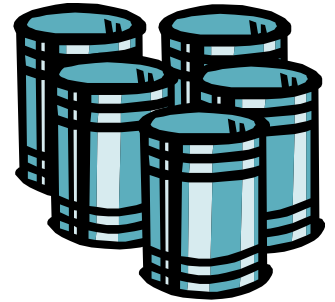
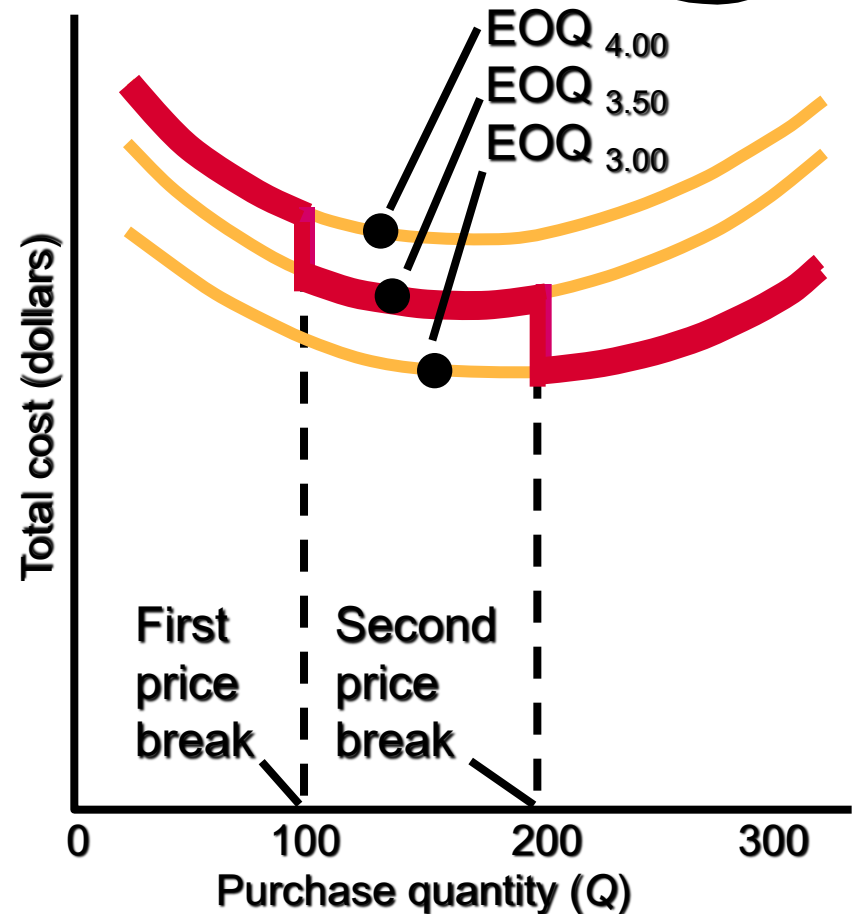
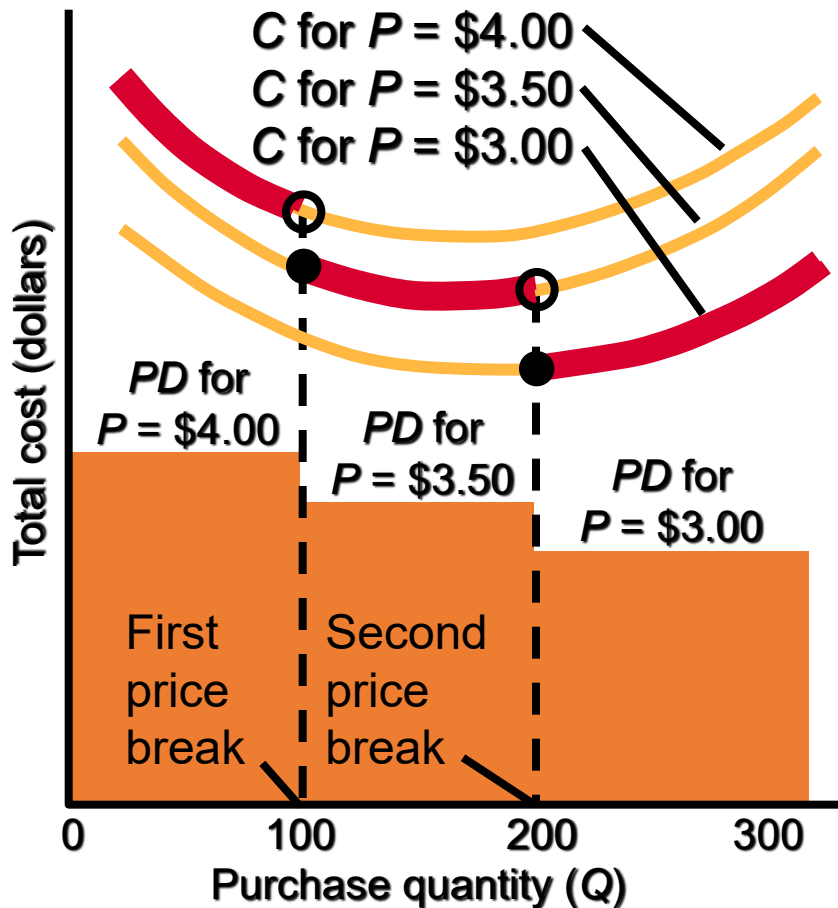


Figure E.3



(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts



Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2DS}{H}}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

$$EOQ_{57.00} = 77 \text{ units}$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

$EOQ_{57.00} = 77 \text{ units}$

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ = 177 units~~

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

EOQ_{\$58.80} = 76 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{57.00} = 77 units~~

EOQ_{58.80} = 76 units

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S) + PD$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \frac{75}{2} [(0.25)(\$60.00)] + \frac{936}{75} (\$45) + \$60.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$C_{75} = \$57,284$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \frac{300}{2} [(0.25)(\$58.80)] + \frac{936}{300} (\$45) + \$58.80(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \frac{500}{2} [(0.25)(\$57.00)] + \frac{936}{500} (\$45) + \$57.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \$56,999$$

Example E.2

Special Inventory Models

Quantity Discounts



Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units

Ordering cost = \$45

Holding cost = 25% of unit price

~~EOQ_{\$57.00} = 77 units~~

~~EOQ_{\$58.80} = 76 units~~

EOQ_{\$60.00} = 75 units

$$C_{75} = \$57,284$$

$$C_{300} = \$57,382$$

$$C_{500} = \$56,999$$

Example E.2

Whole Nature Foods sells a gluten-free product for which the annual demand is 5,000 boxes. At the moment, it is paying \$6.40 for each box; carrying cost is 25% of the unit cost; ordering costs are \$25. A new supplier has offered to sell the same item for \$6.00 if Whole Nature Foods buys at least 3,000 boxes per order. Should the firm stick with the old supplier, or take advantage of the new quantity discount?

Discount

Special Inventory Models

One-Period Decisions



Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10					
20					
30					
40					
50					

For $Q = D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10					
20					
30					
40					
50					

For $Q = D$
Payoff = $(\$10)(10)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100				
20					
30					
40					
50					

For $Q = D$
Payoff = \$100

A red arrow pointing from the text "For Q = D Payoff = \$100" to the cell containing "\$100" in the table.

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20					
30					
40					
50					

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$

$$\text{Payoff} = pD - l(Q - D)$$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$
 Payoff = $(\$10)(30)$
 $- (\$5)(40 - 30)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$

Payoff = $(\$10)(30)$

– $(\$5)(40 - 30)$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$
Payoff = \$250

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20		200	200	200	200
30			300	300	300
40			250	400	400
50					500

For $Q > D$
Payoff = \$250

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = \$10

Loss per ornament after season = \$5

<i>Q</i>	<i>D</i>				
	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

For $Q > D$

$$\text{Payoff} = pD - l(Q - D)$$

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ =

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = 0.2(\$0)

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = $0.2(\$0) + 0.3(\$150)$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = $0.2(\$0) + 0.3(\$150) + 0.3(\$300)$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = $0.2(\$0) + 0.3(\$150) + 0.3(\$300) + 0.1(\$300)$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = $0.2(\$0) + 0.3(\$150) + 0.3(\$300) + 0.1(\$300) + 0.1(\$300)$

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

Expected payoff₃₀ = \$195

	<i>D</i>				
<i>Q</i>	10	20	30	40	50
10	\$100	\$100	\$100	\$100	\$100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions



<i>Demand</i>	10	20	30	40	50
<i>Demand Probability</i>	0.2	0.3	0.3	0.1	0.1

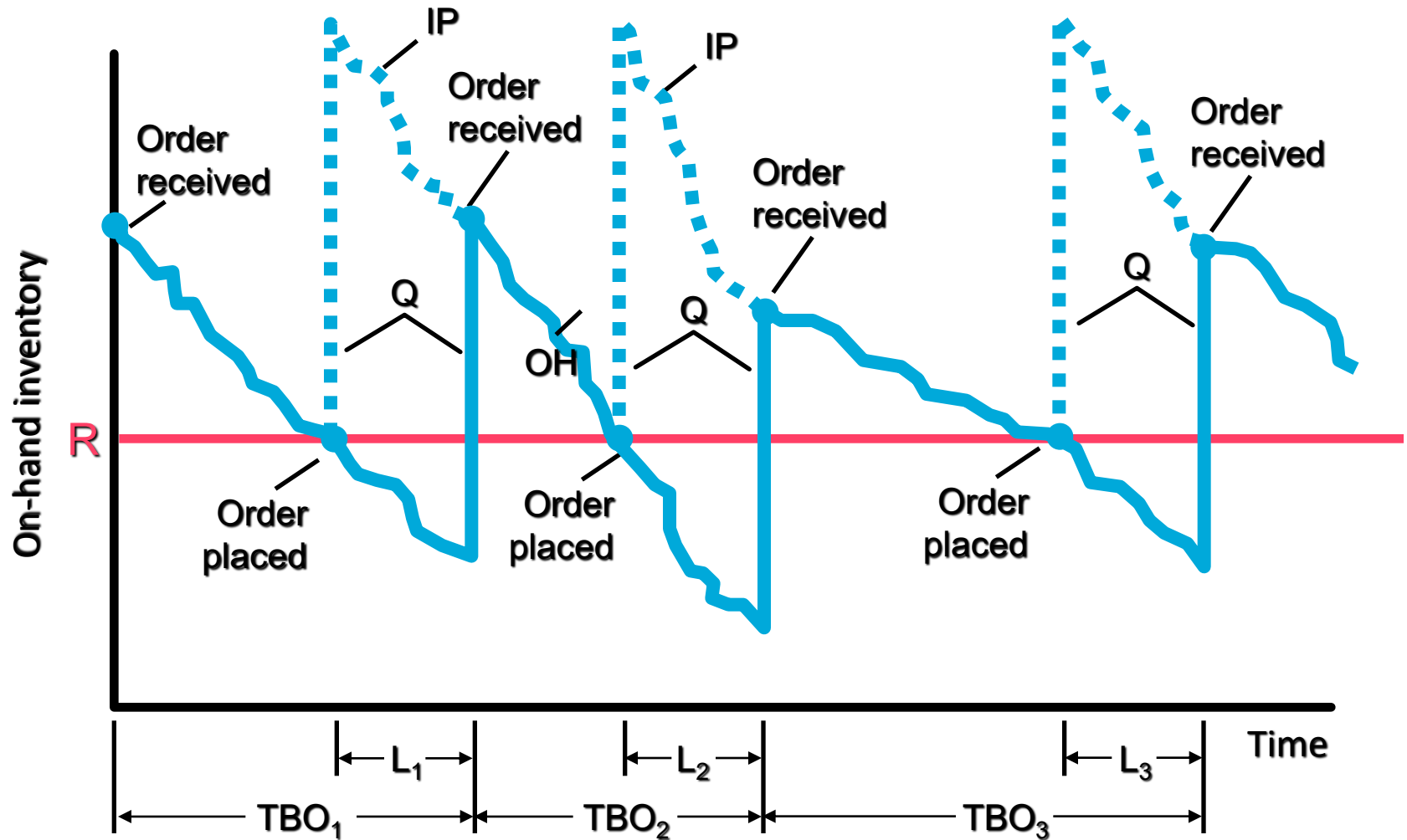
Expected payoff₃₀ = \$195

Q	<i>D</i>					Expected Payoff
	10	20	30	40	50	
10	\$100	\$100	\$100	\$100	\$100	
20	50	200	200	200	200	
30	0	150	300	300	300	195
40	-50	100	250	400	400	
50	-100	50	200	350	500	

Example E.3

Uncertain Demand

Figure 15.8



Reorder Point / Safety Stock

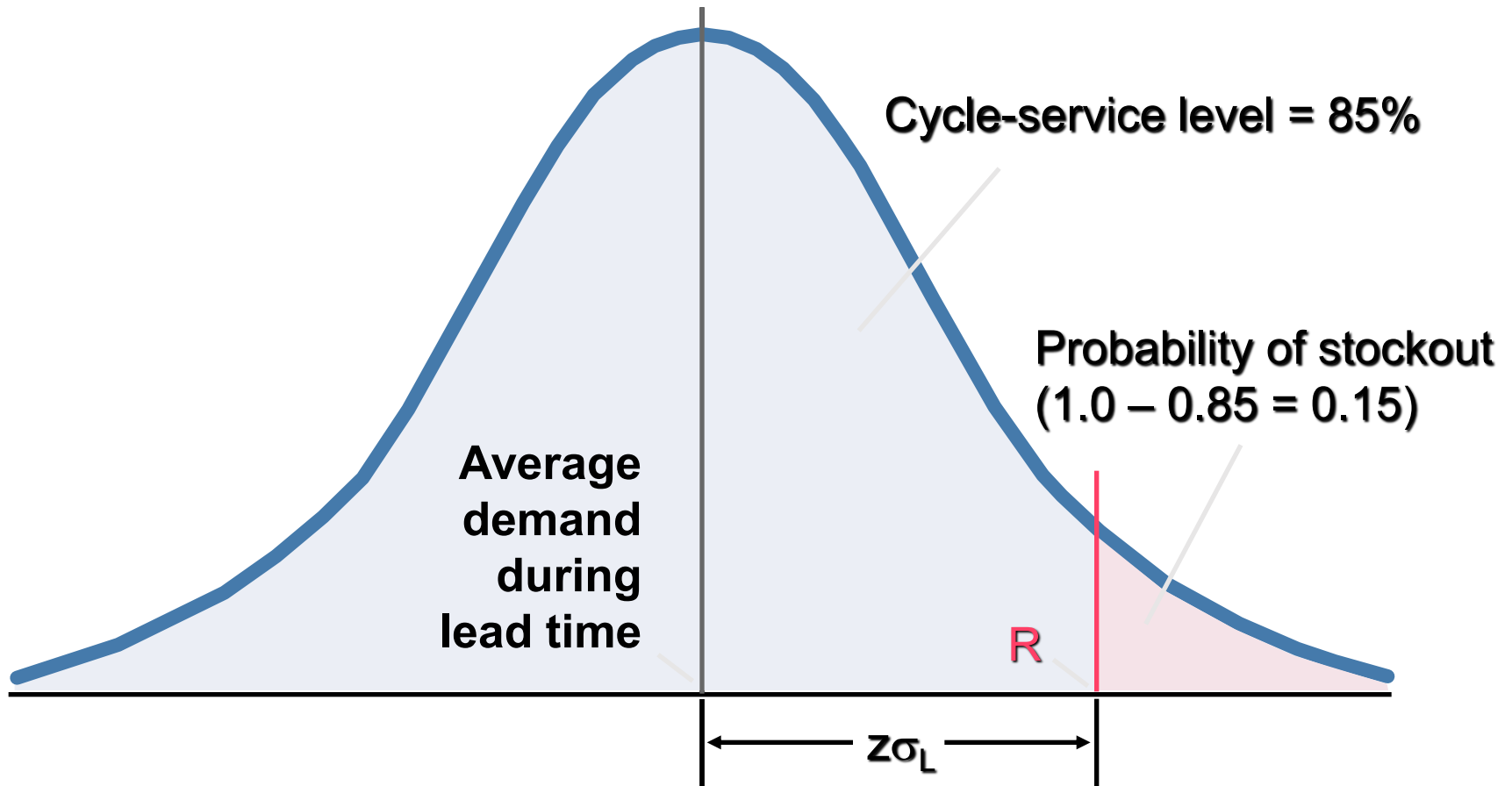


Figure 15.9

EXAMPLE

Records show that the demand for dishwasher detergent during the lead time is normally distributed, with an average of 250 boxes and *variance* $l = 22$.

What safety stock should be carried for a 99 percent cycle-service level?

What is R ?

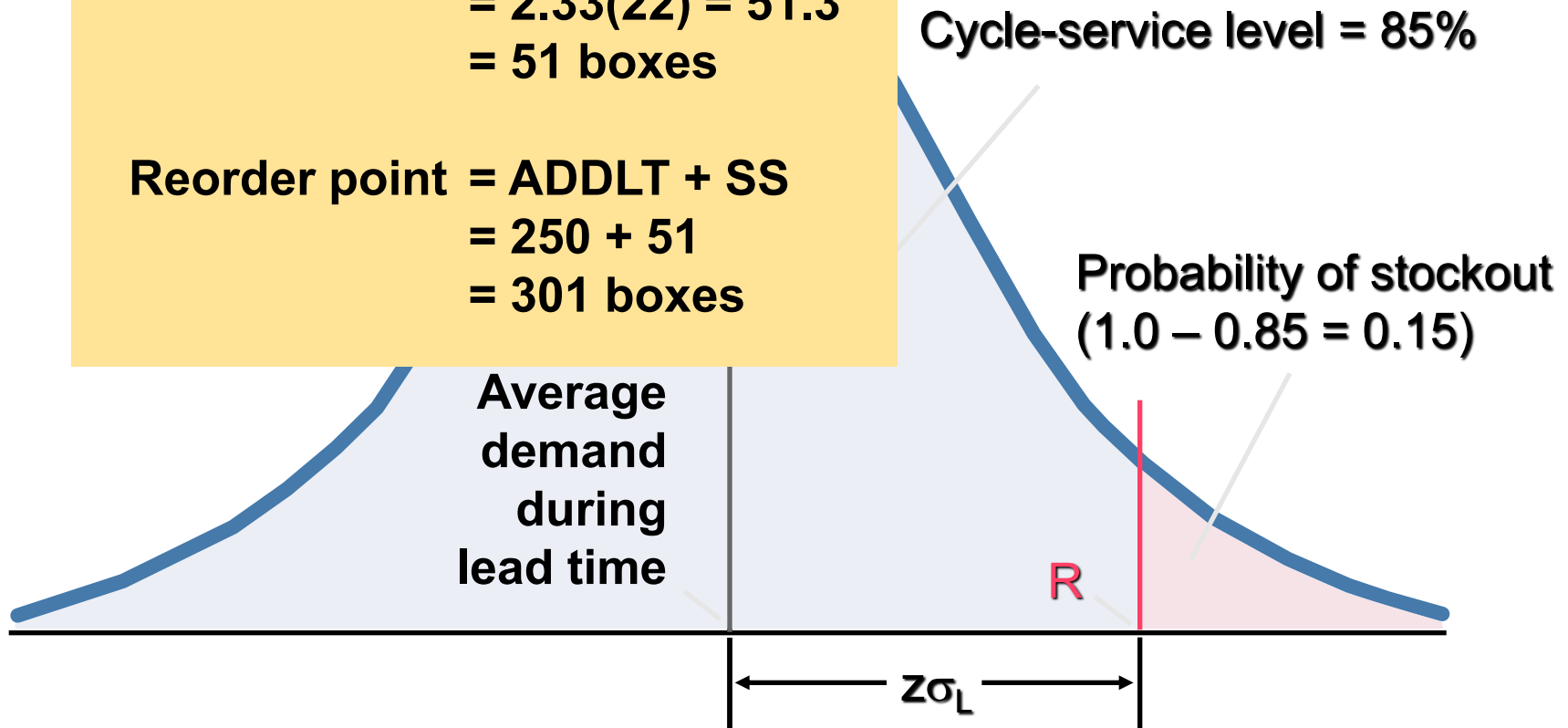
[illegible]

Reorder Point / Safety Stock

Safety Stock/R

$$\begin{aligned}\text{Safety stock} &= z\sigma_L \\ &= 2.33(22) = 51.3 \\ &= 51 \text{ boxes}\end{aligned}$$

$$\begin{aligned}\text{Reorder point} &= \text{ADDLT} + \text{SS} \\ &= 250 + 51 \\ &= 301 \text{ boxes}\end{aligned}$$

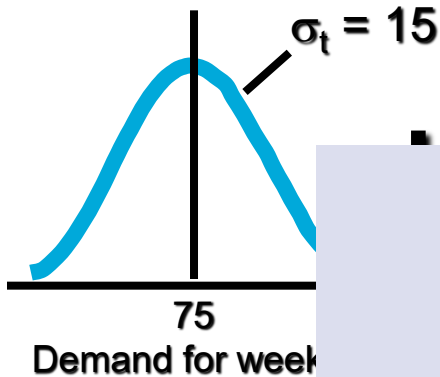


Example Finding the safety stock and R When the Demand Distribution for Lead Time must Be Developed

- Let us return to the bird feeder example. Suppose that the average demand is 18 units per week \
- with a standard deviation of 5 units. The lead time is constant at two weeks. Determine the safety stock and reorder point if management wants a 90 percent cycle-service level. What is the j total cost of the Q system?

[illegible]

Lead Time Distributions



Bird feeder Lead Time Distribution

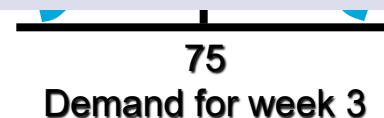
$$t = 1 \text{ week} \quad d = 18 \quad L = 2$$

$$\sigma_L = \sigma_t \sqrt{L} = 15 \sqrt{2} = 21.2$$

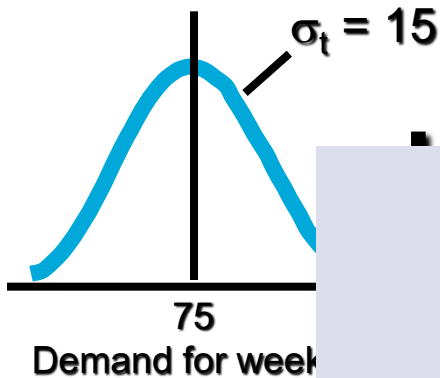
$$\text{Safety stock} = z\sigma_L = 1.28(21.2) = 27.1 \text{ or } 27 \text{ units}$$

$$\begin{aligned} \text{Reorder point} &= dL + \text{Safety stock} \\ &= 2(18) + 27 = 63 \text{ units} \end{aligned}$$

Example 15.6



Lead Time Distributions



Bird feeder Lead Time Distribution

$$t = 1 \text{ week} \quad d = 18 \quad L = 2$$

$$\text{Reorder point} = 2(18) + 9 = 45 \text{ units}$$

$$C = \frac{75}{2}(\$15) + \frac{936}{75}(\$45) + 9(\$15)$$

$$C = \$562.50 + \$561.60 + \$135 = \$1259.10$$

Example 15.6



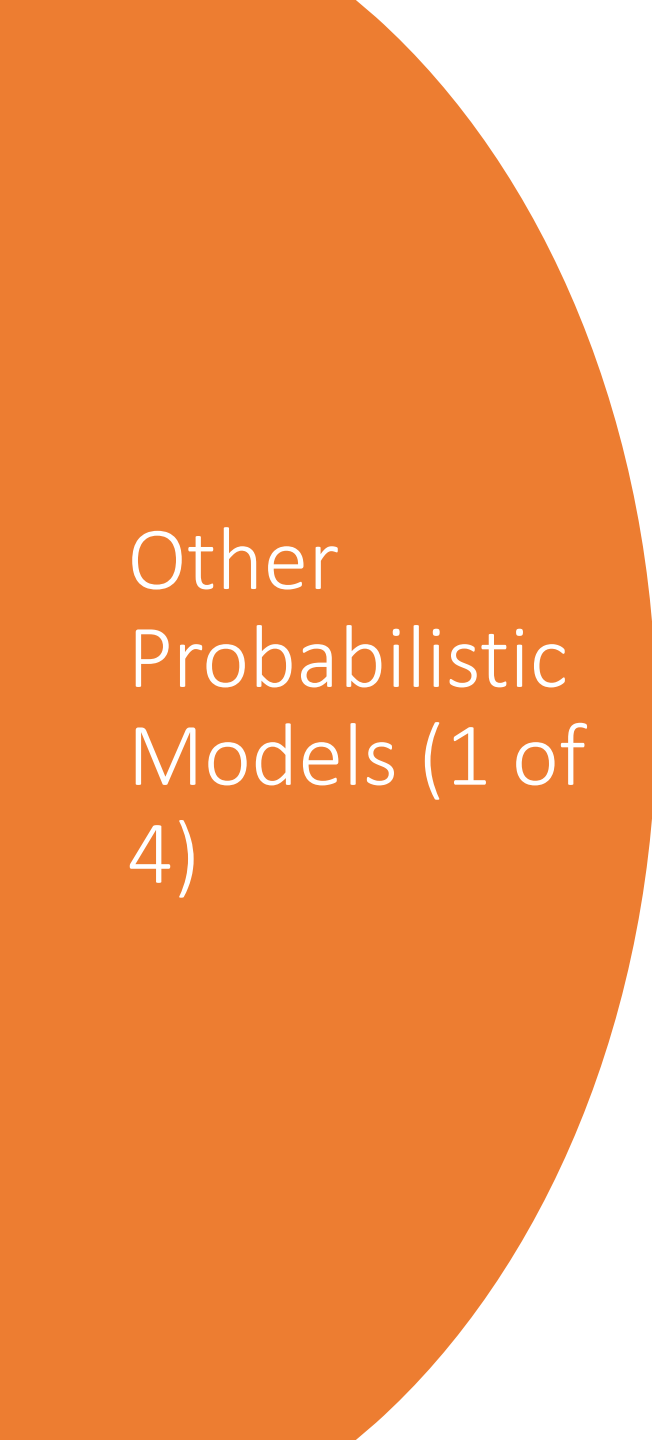
Lead Time Distributions

TABLE 15.1 PROBABILITY DISTRIBUTION FOR LEAD TIME

Lead Time (weeks)	Probability for Lead Time
1	0.35
2	0.45
3	0.10
4	0.05
5	0.05

TABLE 15.2 PROBABILITY DISTRIBUTION FOR DEMAND

Demand (units per week)	Probability of Demand
10	0.10
13	0.20
18	0.40
23	0.20
26	0.10

A large orange circle is positioned on the left side of the slide, partially cut off by the edge.

Other Probabilistic Models (1 of 4)

- When data on demand during lead time are not available, there are other models available
 - When demand is variable and lead time is constant
 - When lead time is variable and demand is constant
 - When both demand and lead time are variable
- 
- A yellow dashed line is located in the bottom right corner of the slide, consisting of several short, curved segments.

Other Probabilistic Models (2 of 4)

Demand is variable and lead time is constant

$$\text{ROP} = (\text{Average daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT}$$

where $\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$

σ_d = Standard deviation of demand per day

Probabilistic Example

Average daily demand (normally distributed) = 15

Lead time in days (constant) = 2

Standard deviation of daily demand = 5

Service level = 90%

Z for 90% = 1.28

From Appendix I

$$\begin{aligned}\text{ROP} &= (15 \text{ units} \times 2 \text{ days}) + Z\sigma_{dLT} \\ &= 30 + 1.28(5)(\sqrt{2}) \\ &= 30 + 9.02 = 39.02 \approx 39\end{aligned}$$

Safety stock is about 9 computers

Gauss table and random distribution Appendix

[illegible]

Other Probabilistic Models (3 of 4)

Lead time is variable and demand is constant

$$\text{ROP} = (\text{Daily demand} \times \text{Average lead time in days}) + Z \times (\text{Daily demand}) \times \sigma_{LT}$$

where σ_{LT} = Standard deviation of lead time in days

Probabilistic Example (3 of 3)

Daily demand (constant) = 10

Average lead time = 6 days

Standard deviation of lead time = $\sigma_{LT} = 1$

Service level = 98%, so Z (from Appendix I) = 2.055

$$\begin{aligned}\text{ROP} &= (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1) \\ &= 60 + 20.55 = 80.55\end{aligned}$$

Reorder point is about 81 cameras

Other Probabilistic Models (4 of 4)

Both demand and lead time are variable

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time}) + Z\sigma_{LT}$$

where σ_d = Standard deviation of demand per day

e σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma_{LT}^2}$$

Probabilistic Example

Average daily demand (normally distributed) = 150

Standard deviation = $\sigma_d = 16$

Average lead time 5 days (normally distributed)

Standard deviation = $\sigma_{LT} = 1$ day

Service level = 95%, so $Z = 1.645$ (from Appendix I)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.645\sigma_{dLT}$$

$$\begin{aligned}\sigma_{dLT} &= \sqrt{(5 \text{ days} \times 16^2) + (150^2 \times 1^2)} = \sqrt{(5 \times 256) + (22,500 \times 1)} \\ &= \sqrt{(1,280) + (22,500)} = \sqrt{23,780} \cong 154\end{aligned}$$

$$\text{ROP} = (150 \times 5) + 1.645(154) \cong 750 + 253 = 1,003 \text{ packs}$$

Single-Period Model

- Only *one* order is placed for a product
- Units have little or no value at the end of the sales period

C_s = Cost of shortage = Sales price/unit – Cost/unit

C_o = Cost of overage = Cost/unit – Salvage value

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single-Period Example (1 of 2)

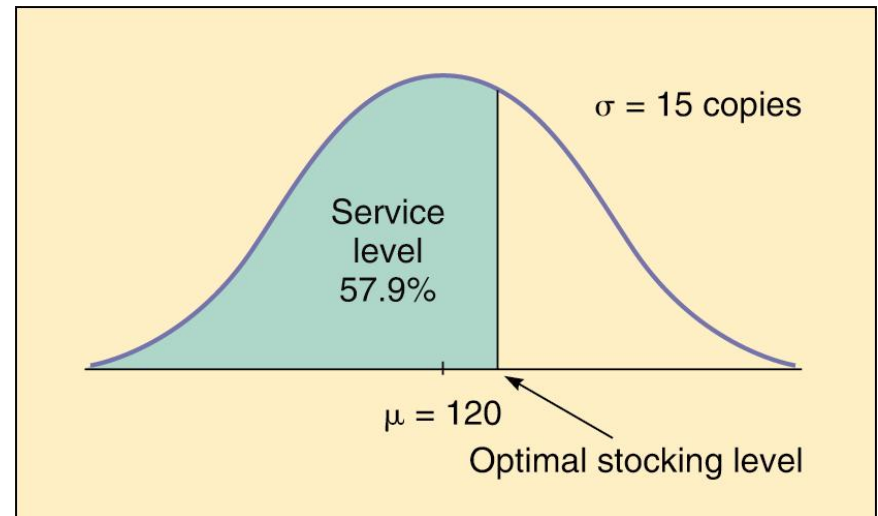
Average demand = $\mu = 120$ papers/day

Standard deviation = $\sigma = 15$ papers

C_s = cost of shortage = $\$1.25 - \$0.70 = \$0.55$

C_o = cost of overage = $\$0.70 - \$0.30 = \$0.40$

$$\begin{aligned}\text{Service level} &= \frac{C_s}{C_s + C_o} \\ &= \frac{.55}{.55 + .40} \\ &= \frac{.55}{.95} = .579\end{aligned}$$



Single-Period Example (2 of 2)

From Appendix I, for the area .579, $Z \cong .199$

The optimal stocking level

$$= 120 \text{ copies} + (.199)(\sigma)$$

$$= 120 + (.199)(15) = 120 + 3 = 123 \text{ papers}$$

The stockout risk = $1 - \text{Service level}$

$$= 1 - .579 = .421 = 42.1\%$$

OTHER QUESTION

- The demand at Arnold Palmer Hospital for a specialized surgery pack is 60 per week, virtually every week.
- The lead time from McKesson, its main supplier, is normally distributed, with a mean of 6 weeks for this product and a standard deviation of 2 weeks.
- A 90% weekly service level is desired.

Find the ROP.

Gauss table and random distribution Appendix

[illegible]

Safety stock

- The inclusion of safety stock (ss) changed the expression to
 - $ROP = d * L + ss$
 - The amount of safety stock maintained depends on the cost of incurring a stockout and the cost of holding the extra inventory. Annual stockout cost is computed as follows:
- Annual stockout costs =
 - The sum of the units short for each demand level * The probability of that demand level * The stockout cost / unit * The number of orders per year

Periodic Review Systems

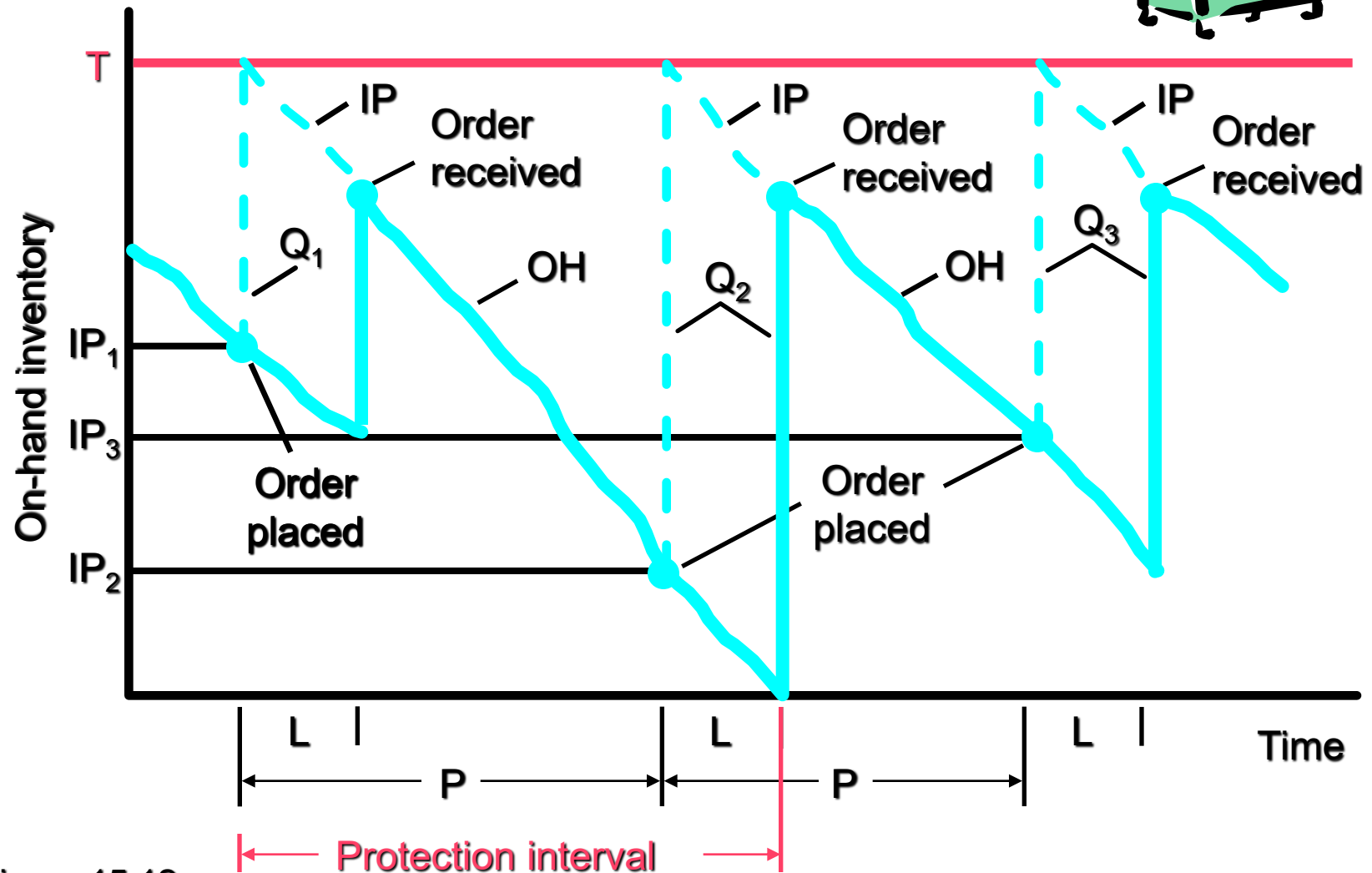
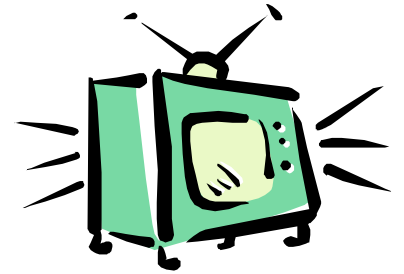
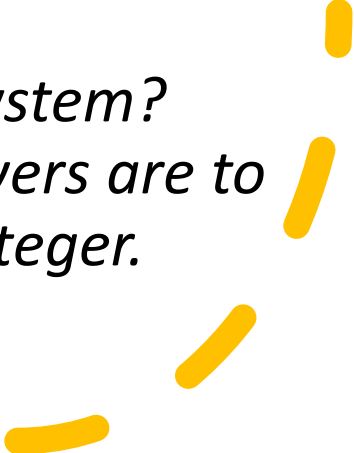


Figure 15.12

Example Calculating P and T

- Again, let us return to the bird feeder example. Recall that demand for the bird feeder is normally distributed with a **mean of 18 units per week** and a **standard deviation in weekly demand of 5 units**. The **lead time is 2 weeks**, and the business operates **52 weeks per year**. The Q system developed in previous example called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent.
 - What is the equivalent *P* system? *What is the total cost ? Answers are to be rounded to the nearest integer.*
- 

Periodic Review Systems



Bird feeder— Calculating P and T

$\sigma_t = 5$ units $L = 2$ weeks cycle/service level = 90%

EOQ = 75 units $D = (18 \text{ units/week})(52 \text{ weeks}) = 936$ units

$$P = \frac{\text{EOQ}}{D} (52) = \frac{75}{936} (52) = 4.2 \text{ or } 4 \text{ weeks}$$

$$\sigma_{P+L} = \sigma_t \sqrt{P + L} = 5 \sqrt{6} = 12 \text{ units}$$

T = Average demand during the protection interval + Safety stock
= $d(P + L) + z\sigma_{P+L}$
= $(18 \text{ units/week})(6 \text{ weeks}) + 1.28(12 \text{ units}) = 123 \text{ units}$

Example 15.8

← Protection interval →

Periodic Review Systems



Bird feeder— Calculating P and T

$\sigma_t = 18$ units $L = 2$ weeks cycle/service level = 90%

EOQ = 75 units $D = (18 \text{ units/week})(52 \text{ weeks}) = 936$ units

$P = 4$ weeks $T = 123$ units

$$C = \frac{4(18)}{2} (\$15) + \frac{936}{4(18)} (\$45) + 15(\$15)$$

$$C = \$540 + \$585 + \$225 = \$1350$$

On-hand inventory

← Protection interval →

Comparison of Q and P Systems

P Systems

- Convenient to administer
- Orders may be combined
- IP only required at review

Q Systems

- Individual review frequencies
- Possible quantity discounts
- Lower, less-expensive safety stocks



QUIZ

INVENTORIES

1-ABC analysis divides on-hand inventory into three classes, based on:

- a) unit price.
- b) the number of units on hand.
- c) annual demand.
- d) annual dollar values.

2-Cycle counting:

- a) provides a measure of inventory turnover.
- b) assumes that all inventory records must be verified with the same frequency.
- c) is a process by which inventory records are periodically verified.
- d) all of the above.

3-The two most important inventory-based questions answered by the typical inventory model are:

- a) when to place an order and the cost of the order.
- b) when to place an order and how much of an item to order.
- c) how much of an item to order and the cost of the order.
- d) how much of an item to order and with whom the order should be placed.

4- Extra units in inventory to help reduce stockouts are called:

- a) reorder point.
- b) safety stock.
- c) just-in-time inventory.
- d) all of the above

5-The difference(s) between the basic EOQ model and the production order quantity model is(are) that:

- A-the production order quantity model does not require the assumption of known, constant demand.
- B-the EOQ model does not require the assumption of negligible lead time.
- C- the production order quantity model does not require the assumption of instantaneous delivery.
- D-all of the above.

6-The EOQ model with quantity discounts attempts to determine:

- A-the lowest amount of inventory necessary to satisfy a certain service level.
- B-the lowest purchase price.
- C-whether to use a fixed-quantity or fixed-period order policy.
- D-how many units should be ordered.
- E- the shortest lead time.

7-The appropriate level of safety stock is typically determined by:

- A-minimizing an expected stockout cost.
- B-choosing the level of safety stock that assures a given service level.
- C-carrying sufficient safety stock so as to eliminate all stockouts.
- D-annual demand

Problem 1

Booker's Book Bindery divides inventory items into three classes, according to their dollar usage. Calculate the usage values of the following inventory items and determine which is most likely to be classified as an A item.

PART NUMBER	DESCRIPTION	QUANTITY USED PER YEAR	UNIT VALUE (\$)
1	Boxes	500	3.00
2	Cardboard (square feet)	18,000	0.02
3	Cover stock	10,000	0.75
4	Glue (gallons)	75	40.00
5	Inside covers	20,000	0.05
6	Reinforcing tape (meters)	3,000	0.15
7	Signatures	150,000	0.45

[illegible]

Problem 2

A regional warehouse purchases hand tools from various suppliers and then distributes them on demand to retailers in the region. The warehouse operates five days per week, 52 weeks per year. Only when it is open can orders be received. The following data are estimated for 3/8-inch hand drills with double insulation and variable speeds:

Average daily demand = 100 drills
Standard deviation of daily demand (σ_d) = 30 drills
Lead time (L) = 3 days
Holding cost (H) = \$9.40/unit/year
Ordering cost (S) = \$35/order
Cycle-service level = 92 percent

The warehouse uses a continuous review (Q) system.

- a. What order quantity, Q and reorder point, R , should be used?
- b. If on-hand inventory is 40 units, there is one open order for 440 drills, and there are no backorders, should a new order be placed?

[illegible]

David Rivera Optical has determined that its reorder point for eyeglass frames is 50 ($d \times L$) units. Its carrying cost per frame per year is \$5, and stockout (or lost sale) cost is \$40 per frame. The store has experienced the following probability distribution for inventory demand during the lead time (reorder period). The optimum number of orders per year is six.

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP $\rightarrow$ 50	.3
60	.2
70	.1
	<u>1.0</u>

How much safety stock should David Rivera keep on hand?

APPROACH ► The objective is to find the amount of safety stock that minimizes the sum of the additional inventory holding costs and stockout costs. The annual holding cost is simply the holding cost per unit multiplied by the units added to the ROP. For example, a safety stock of 20 frames, which implies that the new ROP, with safety stock, is 70 ($- 50 + 20$), raises the annual carrying cost by $\$5(20) = \100 .

However, computing annual stockout cost is more interesting. For any level of safety stock, stockout cost is the expected cost of stocking out. We can compute it, as in Equation (12-12), by multiplying the number of frames short (Demand $-$ ROP) by the probability of demand at that level, by the stockout cost, by the number of times per year the stockout can occur (which in our case is the number of orders per year). Then we add stockout costs for each possible stockout level for a given ROP.⁴

SOLUTION ► We begin by looking at zero safety stock. For this safety stock, a shortage of 10 frames will occur if demand is 60, and a shortage of 20 frames will occur if the demand is 70. Thus the stockout costs for zero safety stock are:

$$\begin{aligned} & (10 \text{ frames short})(.2)(\$40 \text{ per stockout})(6 \text{ possible stockouts per year}) \\ & + (20 \text{ frames short})(.1)(\$40)(6) = \$960 \end{aligned}$$

The following table summarizes the total costs for each of the three alternatives:

SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	$(20)(\$5) = \100	\$ 0	\$100
10	$(10)(\$5) = \$ 50$	$(10)(.1)(\$40)(6) = \240	\$290
0	\$ 0	$(10)(.2)(\$40)(6) + (20)(.1)(\$40)(6) = \$960$	\$960

The safety stock with the lowest total cost is 20 frames. Therefore, this safety stock changes the reorder point to $50 + 20 = 70$ frames.

INSIGHT ► The optical company now knows that a safety stock of 20 frames will be the most economical decision.

Q System

$ROP = d(\text{daily}) \times$
Lead time + safety
stock (= $z \cdot \text{Standard}$
deviation L)

P System

Period=
 EOQ / Annual
demand

$ROP = d \text{ daily}$
 $\times (\text{Period} + \text{Lead}$
time) + safety
stock ($p + l$)